eSpyMath: AP Precalculus

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Precalculus Key Concepts

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. ▷ .001. Polynomial

- A polynomial is an algebraic expression consisting of variables, coefficients, and exponents, combined using addition, subtraction, and multiplication. It can have multiple terms, and the exponents must be non-negative integers.

A **polynomial** is a mathematical expression involving a sum of powers in one or more variables multiplied by coefficients. A single variable polynomial is in the form:

where:

- x is the variable,
- n is a non-negative integer called the degree of the polynomial,
- . $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$. are constants called coefficients, with $a_n \neq 0$ (if $a_n = 0$, the degree is less than n).

The highest power of the variable (x) indicates the **degree** of the polynomial. For **Example**, if the highest power is 3, it's a cubic polynomial; if it's 2, it's a quadratic polynomial; and if it's 1, it's a linear polynomial.

▷ 002. Polynomial Function

A polynomial function is an expression made up of terms called monomials, which are the product of numbers (coefficients) and variables raised to whole number exponents.

- **Example**: $f(x) = 2x^3 - x^2 + 3x - 5$ is a polynomial function.
- **Example**: $f(x) = 5x^5$ is a polynomial function.

▷ 003. Degree of a Polynomial

- The degree of a polynomial is the highest exponent of the variable in the polynomial.
- **Example**: the polynomial $f(x) = 2x^3 - x^2 + 3x - 5$ has a degree of 3.
- **Example**: the polynomial $3x^2 + 2x - 1$ has a degree of 2 because the highest exponent is 2.
- **Example**: The polynomial $7x^5 - 3x^2 + 6$ has a degree of 5.

▹ 004. Leading Coefficient for Polynomials

The leading coefficient is the coefficient of the term with the highest degree.

- In $f(x)=2x^3-x^2+3x-5$, the leading coefficient is 2.
- **Example**: In the polynomial $4x^3-2x^2+5x+1$, the leading coefficient is 4.
- **Example**: In the polynomial $-2x^3+3x-4$, the leading coefficient is -2.

▹ 005. Roots or Zeros for Polynomials

- Roots or zeros of a polynomial are the values of x that make the polynomial equal to zero.
- **Example**: For $x^2-4=0$, the roots are $x=2$ and $x=-2$.

▹ 006. Factoring

Factoring is the process of breaking down a polynomial into simpler polynomials (factors) that, when multiplied together, give the original polynomial.

- **Example**: x^2-4 factors into $(x+2)(x-2)$.
- **Example**: The polynomial x^2-5x+6 can be factored into $(x-2)(x-3)$.

Factoring Formulas

- $a^3-b^3=(a-b)(a^2+ab+b^2)$
- $a^3+b^3=(a+b)(a^2-ab+b^2)$
- $(a^2+2ab+b^2)=(a+b)^2$
- $(a^2-2ab+b^2)=(a-b)^2$
- $(a^3-3a^2b+3ab^2-b^3)=(a-b)^3$
- $(a^3+3a^2b+3ab^2+b^2)=(a+b)^3$
- $a^2-b^2=(a-b)(a+b)$

- $\left(\sqrt{a}+\sqrt{b}\right)^2 = a + 2\sqrt{ab} + b = (a+b) + 2\sqrt{ab}$
- $\left(\sqrt{a}-\sqrt{b}\right)^2 = a - 2\sqrt{ab} + b = (a+b) - 2\sqrt{ab}$
- $\sqrt{(a+b)+2\sqrt{ab}} = \sqrt{a} + \sqrt{b}$
- $\sqrt{(a+b)-2\sqrt{ab}} = \sqrt{a} - \sqrt{b}\ (a \geq b)$
- $\left(a^2+b^2+c^2+2ab+2bc+2ac\right) = (a+b+c)^2$

▷ 007. Factor Theorem

The Factor Theorem states that $x-c$ is a factor of a polynomial $f(x)$ if and only if $f(c)=0$.

- **Example**: If $f(2)=0$ for $f(x)=2x^3-4x^2+4x-8$, then $x-2$ is a factor of $f(x)$.
- **Example**: If $f(3)=0$ for $f(x)=x^3-9x^2+27x-27$, then $(x-3)$ is a factor.

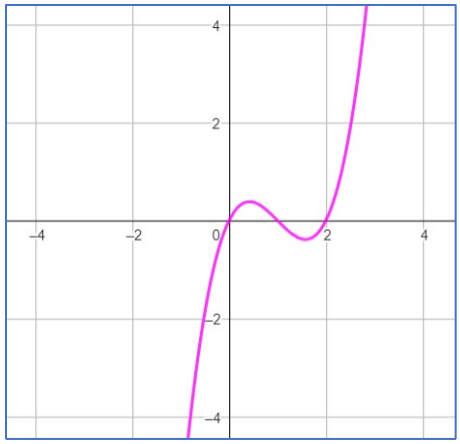
	2	-4	4	-8
2		4	0	8
	2	**0**	**4**	**0**

	1	-9	27	-27
3		3	-18	27
	1	**-6**	**9**	**0**
3		0	-9	
	1	-3	**0**	

Factor Theorem:

- The Factor Theorem is a special case of the Remainder Theorem. It states that $x-a$ is a factor of the polynomial $f(x)$ if and only if $f(a)=0$.
- In summary, the Remainder Theorem helps you find the remainder when a polynomial is divided by a linear expression, and the Factor Theorem helps you determine whether that linear expression is a factor of the polynomial. Both theorems are extremely useful for simplifying polynomial division and for finding the roots of polynomials.

- **Example:** Let's say we have a polynomial $f(x)=x^2-3x+2$, and we want to divide it by $x-1$. According to the Remainder Theorem, the remainder of this division is simply $f(1)$. So, we evaluate $f(x)$ at $x=1$: $f(1)=(1)^2-3(1)+2=1-3+2=0$
- Solution: The remainder is 0, which also tells us that $x-1$ is a factor of $f(x)$.



▷ 034. Continuity

A function is continuous at a point if there is no interruption in the graph at that point. The function's limit, as it approaches the point, must equal the function's value at that point.

- **Example**: The function $f(x)=x^2$ is continuous at $x=2$ because $\lim_{x\to 2} x^2=4$ and $f(2)=4$.

▷ 035. Extrema, Intervals of Increase and Decrease

Extrema refers to the maximum and minimum values of a function. A function is increasing when the output values increase as the input values increase. It is decreasing when the output values decrease as the input values increase.

- **Example**: For $f(x)=x^2-4x$, $f(x)$ is decreasing on the interval $(-\infty,2)$ and increasing on the interval $(2,\infty)$. The point $x=2$ is a minimum (extrema).

▷ 036. Power Functions

Power functions have the form $f(x)=ax^n$ where n is a real number.

- **Example**: $f(x)=2x^3$ is a power function with a degree of 3.

▷ 037. Average Rates of Change

The average rate of change of a function over an interval is the change in the function's value divided by the change in the input value.

The rate of change is the speed at which a function's output changes with respect to its input. For a function $f(x)$, the **average rate of change** from $x = a$ to $x = b$ is given by $\frac{f(b)-f(a)}{b-a}$.

- **Example**: The average rate of change of $f(x) = x^2$ from $x = 1$ to $x = 3$ is $\frac{f(3)-f(1)}{3-1} = \frac{9-1}{2} = 4$.

Rates of Change in Linear and Quadratic Functions:

For **linear functions** of the form $f(x) = mx + b$, the rate of change is constant and equal to the slope m. This means that for any two points on the line, the rate of change (or slope) between them is the same. For example, if $f(x) = 3x + 2$, the rate of change is 3, indicating that for every one unit increase in x, $f(x)$ increases by 3 units.

For **quadratic functions** of the form $f(x) = ax^2 + bx + c$, the rate of change is not constant and is represented by the difference in function values over the difference in x values. The rate of change increases or decreases as x moves away from the vertex of the parabola. Algebraically, the average rate of change from x_1 to x_2 is

$$\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{a(x_2^2-x_1^2)+b(x_2-x_1)}{x_2-x_1} = 2ax + b.$$

Polynomial Functions and Rates of Change:

The rate of change of a polynomial function can be explored by looking at the differences in the function's values over intervals. For a polynomial function $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, the rate of change between two points can be found by calculating the average rate of change as described above. For example, the average rate of change of $f(x) = x^3 - 4x$ between $x = 1$ and $x = 3$ is $\frac{f(3)-f(1)}{3-1}$. This gives us an idea of how steeply the function is increasing or decreasing over that interval. As the degree of the polynomial increases, the behavior of the rates of change becomes more complex, with more turning points and inflection points.

▷ 038. Transformations of Graphs

Transformations include shifting, reflecting, stretching, or compressing the graph of a function.

- **Example**: The graph of $f(x)=(x-2)^2$ is a horizontal shift to the right by 2 units of the graph of $g(x)=x^2$.
- These transformations can be combined to move and reshape the graph of a polynomial function in various ways, providing a powerful tool for understanding and manipulating the function's graphical representation.

Shifting

- **Vertical Shift**: Adding or subtracting a constant to the function moves the graph up or down.
- **Example**: $f(x)=x^2$ shifted up by 3 units becomes $f(x)=x^2+3$.

- **Horizontal Shift**: Adding or subtracting a constant inside the function argument moves the graph left or right.
- **Example**: $f(x)=x^2$ shifted right by 2 units becomes $f(x)=(x-2)^2$.

Reflecting

- **Reflect Across the x-axis**: Multiplying the function by -1 reflects it across the x-axis.
- **Example**: $f(x)=x^2$ reflected across the x-axis becomes $f(x)=-x^2$.

- **Reflect Across the y-axis**: Replacing x with $-x$ in the function reflects it across the y-axis.
- **Example**: $f(x)=x^3$ reflected across the y-axis becomes $f(x)=(-x)^3=-x^3$.

Stretching or Compressing

- **Vertical Stretch**: Multiplying the function by a constant greater than 1 stretches it vertically.
- **Example**: $f(x)=x^2$ stretched vertically by a factor of 2 becomes $f(x)=2x^2$.

- **Vertical Compression**: Multiplying the function by a constant between 0 and 1 compresses it vertically.
- **Example**: $f(x)=x^2$ compressed vertically by a factor of 0.5 becomes $f(x)=0.5x^2$.

- **Horizontal Stretch**: Multiplying the variable by a constant between 0 and 1 stretches the graph horizontally.
- **Example**: $f(x)=x^2$ stretched horizontally by a factor of 0.5 becomes $f(x)=(0.5x)^2=0.25x^2$.

- **Horizontal Compression**: Multiplying the variable by a constant greater than 1 compresses the graph horizontally.
- **Example**: $f(x)=x^2$ compressed horizontally by a factor of 2 becomes $f(x)=(2x)^2=4x^2$.

▹039. Piecewise Functions

- **Hybrid functions**, also known as **piecewise functions**, are defined by different expressions over different intervals within their domain. Each piece of the function applies to a specific part of the domain.
- **Example**: $f(x)=\begin{cases} x^2 & \text{if } x<0 \\ x+2 & \text{if } x\geq 0 \end{cases}$.

Graphing a Hybrid Function

When graphing a hybrid function, you plot each piece separately within its specified interval. The graph may have **sharp turns** or **jumps** at the boundaries where the function's rule changes.

Domain and Range of a Hybrid Function

The domain of a hybrid function is the set of all x values for which the function is defined, which may be all real numbers or a subset, depending on the pieces. The range is the set of all possible output values.

Continuity of a Hybrid Function

A hybrid function may or may not be continuous. It is continuous if you can draw the graph without lifting your pencil from the paper. Discontinuities can occur at the boundaries between pieces.

▷ 040. Operations on functions

Operations on functions include addition, subtraction, multiplication, and division of two functions.

- When performing these operations, it's important to consider the domains of the original functions to ensure the operations are defined. The domain of the resulting function may be restricted based on the need to avoid division by zero or other undefined expressions.

Addition of Functions

- Combining two functions by adding their corresponding output values.
- **Example**: If $f(x)=x^2$ and $g(x)=3x+1$, then $(f+g)(x)=f(x)+g(x)=x^2+(3x+1)$.

Subtraction of Functions

- Combining two functions by subtracting the output values of one function from the other.
- **Example**: If $f(x)=x^2$ and $g(x)=3x+1$, then $(f-g)(x)=f(x)-g(x)=x^2-(3x+1)$.

Multiplication of Functions

- Combining two functions by multiplying their output values.
- **Example**: If $f(x)=x^2$ and $g(x)=3x+1$, then $(f\cdot g)(x)=f(x)\cdot g(x)=x^2\cdot(3x+1)$.

Division of Functions

- Combining two functions by dividing the output value of one function by the output value of the other if the denominator is not zero.
- **Example**: If $f(x)=x^2$ and $g(x)=3x+1$, then $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{x^2}{3x+1}$, for $x\neq-\frac{1}{3}$.

▷ 041. Inverse function

The inverse of a function reverses the original function's effect. If $f(x)$ maps a to b, then $f^{-1}(x)$ maps b to a.

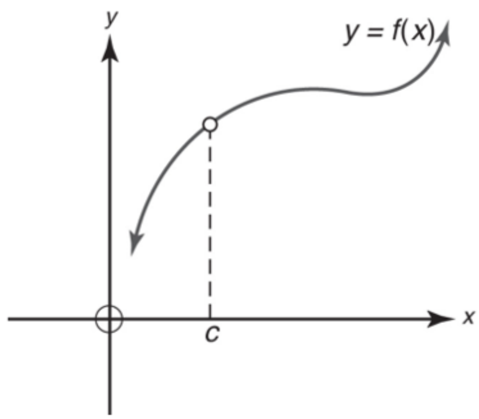
- **Example**: If $f(x)=2x+3$, its inverse is $f^{-1}(x)=\frac{x-3}{2}$.

- **Example**: If $f(x)=10^x$, its inverse is $f^{-1}(x)=\log x$.

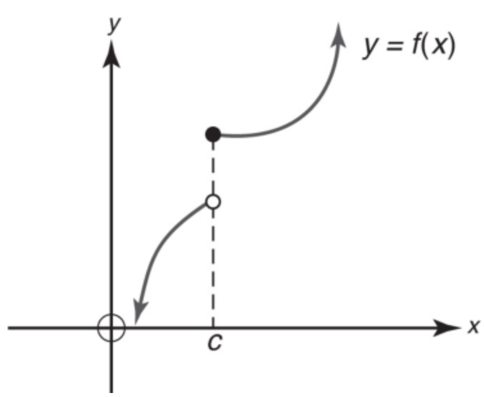
▹042. Types of discontinuities

A function is discontinuous when it doesn't meet the criteria for being continuous at a certain point. Here are the types of discontinuities:

- **Hole or Point Discontinuity**: This occurs when the function is not defined at a particular point x = c, yet the limits from both sides as x approaches c are equal. On a graph, this is shown as an open circle x = c.



- **Jump Discontinuity:** If the function's limits from the left and right as x approaches c exist but are not the same, the function has a "jump" at x = c. This is seen as a sudden leap or gap in the graph.



- **Infinite Discontinuity:** When the function's limit as x approaches c does not exist because the function heads towards infinity, there is an infinite discontinuity. This is typically represented by a vertical asymptote on the graph at x = c .

- The standard form of a hyperbola's equation is $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$.
- A hyperbola is the set of all points for which the difference of the distances from two fixed points (foci) is constant. The standard form of a hyperbola's equation with the center at the origin is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ for a horizontal hyperbola or $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ for a vertical hyperbola.
- Foci $a^2+b^2=c^2$, Asymptotes ($y=\pm\frac{a}{b}$), Eccentricity ($\frac{c}{a}$), Parametric Equations $x=a\cosh(t)$, $y=b\sinh(t)$ where t is the parameter.

- $\frac{x^2}{9}-\frac{y^2}{4}=1$ (Horizontal hyperbola centered at the origin)
- $\frac{y^2}{16}-\frac{x^2}{9}=1$ (Vertical hyperbola centered at the origin)
- $\frac{(x-3)^2}{9}-\frac{(y+2)^2}{4}=1$ (Horizontal hyperbola centered at (3, -2))
- $\frac{(y-5)^2}{16}-\frac{(x+4)^2}{9}=1$ (Vertical hyperbola centered at (-4, 5))

▷ 116. Limits

- The concept of a limit examines the behavior of a function as it approaches a particular point.
- **Example**: $\lim_{x\to 2}\frac{x^2-4}{x-2}=4$.

▷ 117. Continuity

- A function is continuous at a point if the limit as it approaches the point equals the function's value at that point.
- **Example**: The function $f(x)=x^2$ is continuous at $x=2$ because $\lim_{x\to 2}x^2=4$ and $f(2)=4$.

▷ 118. Exponential Growth and Decay

- Models that describe how quantities grow or shrink over time.
- **Example**: The population P over time t with growth rate r is $P(t) = P_0 e^{rt}$.

▷ 119. Logarithmic Scale

- A scale used to represent data that spans multiple orders of magnitude.
- **Example**: The Richter scale for earthquake intensity is logarithmic.

▷ 120. Rationalizing Substitutions

- A technique used to simplify integrals and solve equations by substituting a trigonometric or hyperbolic function.
- **Example**: Substitute $x = \tan(\theta)$ to rationalize $\sqrt{x^2+1}$.

▷ 121. Parent Function

Parent functions are the simplest forms of functions within their family.

They provide the basic shape or template from which more complicated functions in the same family can be derived by transformations such as shifts, stretches, compressions, and reflections. here are some common parent functions in precalculus:

Linear function

- Parent function: $f(x) = x$
- **Example**: $g(x) = 2x + 3$ is a linear function derived from the parent function by stretching and shifting.

Quadratic function

- Parent function: $f(x) = x^2$
- Graph: a parabola opening upwards with its vertex at the origin.
- **Example**: $g(x) = (x-2)^2 - 3$ is a quadratic function derived from the parent function by shifting to the right and down.

Cubic function

- Parent function: $f(x)=x^3$
- Graph: an s-shaped curve passing through the origin.
- **Example**: $g(x)=-x^3$ is a cubic function derived from the parent function by reflecting it across the x-axis.

Absolute value function

- Parent function: $f(x)=|x|$
- Graph: a v-shaped graph with its vertex at the origin.
- **Example**: $g(x)=2|x-1|+4$ is an absolute value function derived from the parent function by stretching, shifting to the right, and up.

Square root function

- Parent function: $f(x)=\sqrt{x}$
- Graph: half of a sideways parabola opening to the right.
- **Example**: $g(x)=\sqrt{x+3}-2$ is a square root function derived from the parent function by shifting to the left and down.

Exponential function

- Parent function: $f(x)=e^x$ or $f(x)=a^x$ where $a>0$
- Graph: a rapidly increasing curve passing through the point (0,1).
- **Example**: $g(x)=2^x$ is an exponential function with base 2, derived from the parent function with base 'e'.

Trigonometric functions

- Parent sine function: $f(x)=\sin(x)$
- Graph: a wave that oscillates between -1 and 1 with a period of 2π.
- **Example**: $g(x)=3\sin(2x)$ is a sine function derived from the parent function by stretching and changing the period.

- Parent cosine function: $f(x)=\cos(x)$
- Graph: similar to the sine function but starts at its maximum value of 1.

- **Example**: $g(x) = \cos(x + \frac{\pi}{4})$ is a cosine function derived from the parent function by shifting to the left.

▷ 122. Logarithmic function

- Parent function: $f(x) = \log(x)$ or $f(x) = log_a(x)$ where $a > 0$ and $a \neq 1$
- Graph: a slowly increasing curve passing through the point (1, 0).
- **Example**: $g(x) = log_2(x-1)$ is a logarithmic function derived from the parent function by shifting to the right.

Evaluating Logarithms

- To evaluate a logarithm, determine the exponent that the base must be raised to in order to obtain the number.
- **Example**: Evaluate $\log_2(32)$. Since $2^5 = 32$, $\log_2(32) = 5$.

Logarithms and Exponents as Inverses

- Logarithms undo the effect of exponents, and vice versa.
- **Example**: If $e^x = y$, then $\ln(y) = x$.

Properties of Logarithms

- Logarithms have properties that simplify the manipulation of logarithmic expressions, such as the product, quotient, and power rules.
- **Example**: $\log_b(MN) = \log_b(M) + \log_b(N)$.

Writing Logs in Terms of Others

- Logarithms can be expressed in terms of logarithms with different bases using change-of-base formulas.
- **Example**: $\log_2(8)$ can be written as $\frac{\log(8)}{\log(2)}$.

Exponential Equations Requiring Logarithms

- To solve exponential equations where the same base cannot be used, apply logarithms to both sides of the equation.

- **Example**: Solve $2^x = 5$ by taking the natural logarithm of both sides: $x = \ln(5)/\ln(2)$.

Logarithmic Equations

- Equations that can be solved by applying the properties of logarithms to isolate the variable.
- **Example**: Solve $\log(x) + \log(2) = 3$ by combining logs: $\log(2x) = 3$, then $2x = 10^3$.
- **Example**: Solve $\log_2(x^2 - 6) = 3$ by converting to exponential form: $x^2 - 6 = 2^3$, then $x^2 = 14$.

Graphing Logarithmic Functions

- Logarithmic functions have the form $f(x) = \log_b(x)$ and their graphs are the inverse of exponential functions.
- **Example**: Graph $f(x) = \log_2(x)$. The graph passes through $(1,0)$ and increases slowly for positive x.

Compound Interest

- Compound interest is calculated using the formula $A = P(1 + \frac{r}{n})^{nt}$, where P is the principal, r is the annual interest rate, n is the number of times interest is compounded per year, and t is the time in years.
- **Example**: Calculate the amount of a \$1000 investment after 3 years at an annual interest rate of 5% compounded quarterly: $A = 1000(1 + \frac{0.05}{4})^{4\times3}$.

▷ 123. Polar form and rectangular form for complex number

Rectangular Form:

- In rectangular form, a complex number is represented as a combination of a real part and an imaginary part, using the format $a + bi$, where (a) is the real part, (b) is the imaginary part, and (i) is the imaginary unit with the property that $i^2 = -1$.

Polar Form:

- In polar form, a complex number is represented using a magnitude (also called modulus or absolute value) and an angle (also called argument or phase), using the format

$r(\cos(\theta)+i\sin(\theta))$ *or* $rCis(\theta)$ or $r\angle\theta$, where ® is the magnitude and (θ) is the angle in radians.

- Rectangular form: $z=a+bi$
- Polar form: $z=r\ \cos\theta+(r\ \sin\theta)i=r(\cos\theta+i\ \sin\theta)=r\,cis\,\theta$

Conversion from Rectangular to Polar Form:

- To convert a complex number from rectangular to polar form, you calculate the magnitude r and the angle θ using the following equations:

$$r=\pm\sqrt{a^2+b^2},\ x=r\cos(\theta),\ y=r\sin(\theta)$$

$$r=|z|=\sqrt{a^2+b^2}=\sqrt{(rcos\,\theta)^2+(rsin\,\theta)^2}$$

$\tan\theta=\left(\frac{b}{a}\right)$ or use $\theta=\arctan\left(\frac{b}{a}\right)$ for the correct quadrant.

Conversion from Polar to Rectangular Form:

- To convert a complex number from polar to rectangular form, you use the magnitude (r) and the angle (θ) to find the real part (a) and the imaginary part (b) using the following equations:

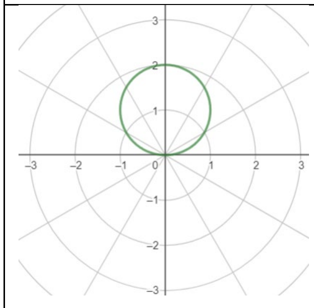
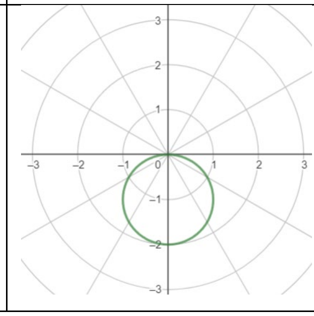
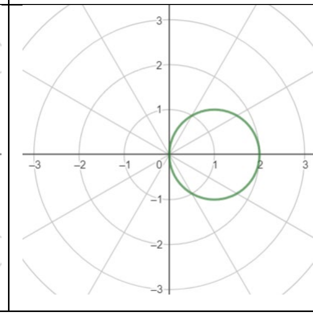
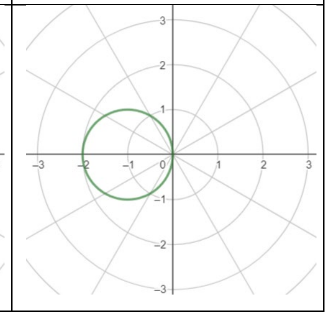
▷ 124. Change in Tandem

This concept involves understanding how two or more related quantities **vary together**. For example, if the volume of a cube changes, its surface area changes as well.

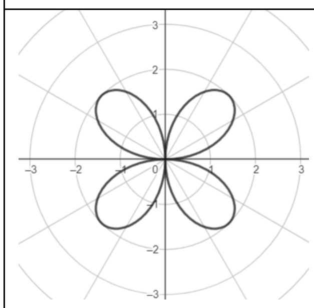
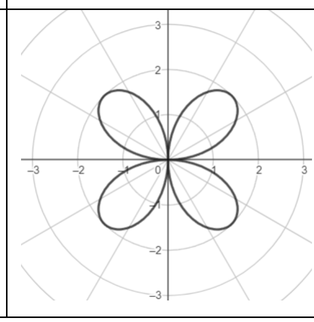
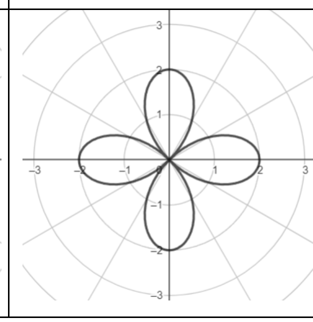
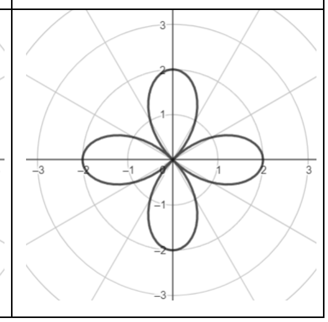
Examples:

1) The area and perimeter of a square increase together as the side length increases.
2) The volume and surface area of a cube change as the side length changes. If the side length doubles, the volume increases eightfold, while the surface area increases fourfold.
3) The relationship between the radius and the circumference of a circle. As the radius changes, the circumference changes proportionally, with $C=2\pi r$.
4) The volume and surface area of a sphere ($V=\frac{4}{3}\pi r^3$ and $A=4\pi r^2$) - as the radius increases, both the volume and surface area increase.

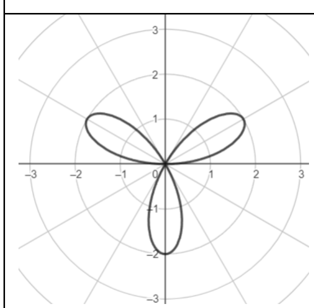
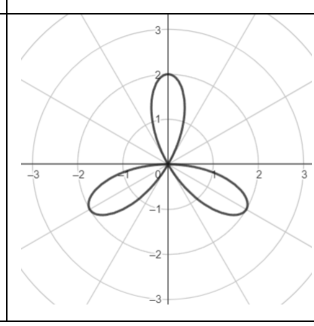
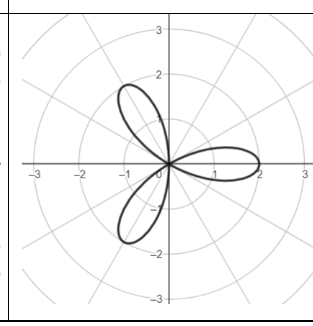
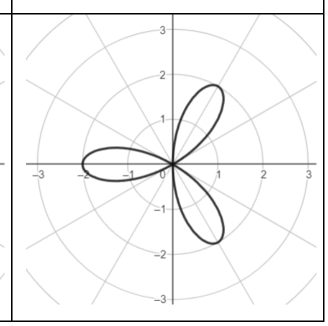
▷ 125. Polar Graphs

$r(\theta)=2\sin(\theta)$	$r(\theta)=-2\sin(\theta)$	$r(\theta)=2\cos(\theta)$	$r(\theta)=-2\cos(\theta)$
			

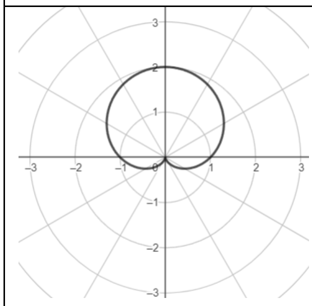
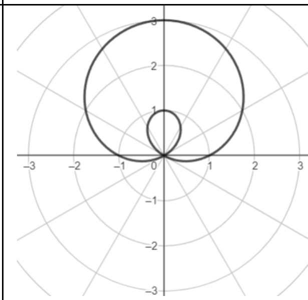
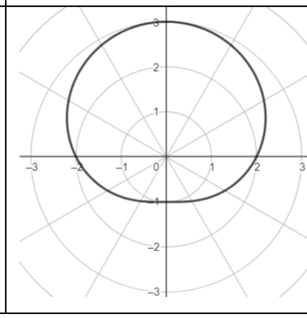
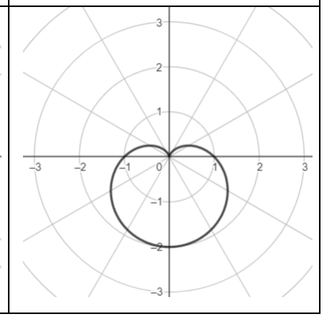
$r(\theta)=a\sin(b\theta)$ or $r(\theta)=a\cos(b\theta)$: if b= even, the polar curve will have 2b petals.

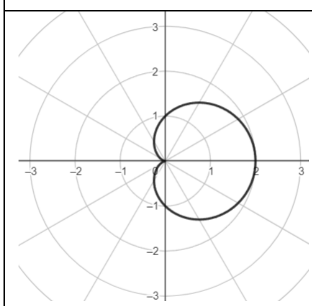
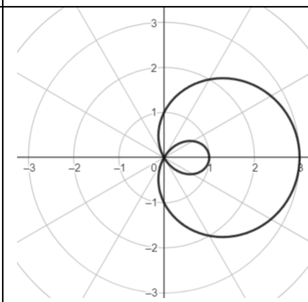
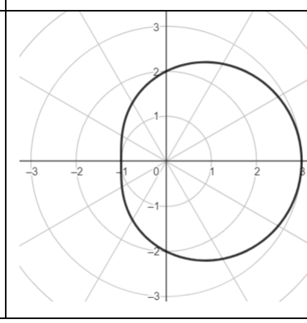
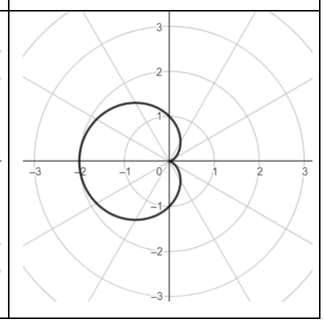
$r(\theta)=2\sin(2\theta)$	$r(\theta)=-2\sin(2\theta)$	$r(\theta)=2\cos(2\theta)$	$r(\theta)=-2\cos(2\theta)$
			

$r(\theta)=a\sin(b\theta)$ or $r(\theta)=a\cos(b\theta)$: if b= odd, the polar curve will have b petals.

$r(\theta)=2\sin(3\theta)$	$r(\theta)=-2\sin(3\theta)$	$r(\theta)=2\cos(3\theta)$	$r(\theta)=-2\cos(3\theta)$
			

$r(\theta)=a\pm b\sin(\theta)$ or $r(\theta)=a\pm b\cos(\theta)$

$r(\theta)=1+\sin(\theta)$ Cardoid: $\frac{a}{b}<1$	$r(\theta)=1+2\sin(\theta)$ Limacon: $\frac{a}{b}=1$	$r(\theta)=2+\sin(\theta)$ Dimpled Cardoid: $1<\frac{a}{b}<2$	$r(\theta)=1-\sin(\theta)$
			

$r(\theta)=1+\cos(\theta)$ Cardoid: $\frac{a}{b}<1$	$r(\theta)=1+2\cos(\theta)$ Limacon: $\frac{a}{b}=1$	$r(\theta)=2+\cos(\theta)$ Dimpled Cardoid: $1<\frac{a}{b}<2$	$r(\theta)=1-\cos(\theta)$
			

Chapter 1. Polynomial and Rational functions

#1-01. Definition of Polynomial

Is the equation a polynomial? (True or False)

1) $y = \pi x^2 + 3x + 4$	2) $y = \pi x^{1.2}$
3) $y = \pi x^{1/2} + 4x$	4) $y = \pi\sqrt{x^2}$
5) $y = x + x^{1/2}$	6) $y = \lvert x \rvert$

Solutions:

1) $\Rightarrow$ **True** $y = \pi x^2 + 3x + 4$: **True**. The exponents are whole numbers and the coefficients are real numbers.	**2)** $\Rightarrow$ **False** $y = \pi x^{1.2}$: **False**. The exponent is not a whole number.
3) $\Rightarrow$ **False** $y = \pi x^{1/2} + 4x$: **False**. The exponent $1/2$ is not a whole number.	**4)** $\Rightarrow$ **False** $y = \pi\sqrt{x^2}$: **False**. This simplifies to $y = \pi \| x \|$, which is not a polynomial due to the absolute value. However, if we consider x^2 under the square root, it is a whole number exponent, but due to the absolute value, the overall answer is False.
5) $\Rightarrow$ **False** $y = x + x^{1/2}$: **False**. The exponent $1/2$ is not a whole number.	**6)** $\Rightarrow$ **False** $y = \| x \|$: **False**. A polynomial function must have exponents that are whole numbers, and the absolute value function does not satisfy this condition.

#1-02. Exploring Different Types of Algebraic Functions

Solve each equation for x.

1) $y = ax + b$	2) $y = ax^2 - b$
3) $y = ax^3 - b$	4) $y = x^2 - 2x + 3$
5) $y = x^{-1} - (x+1)^{-1}$	6) $y = \dfrac{x+1}{2x-3}$

Solutions:

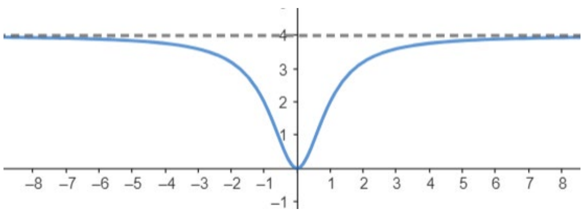
1) $\Rightarrow x=\dfrac{y-b}{a}$ $y=ax+b$: • $x=\dfrac{y-b}{a}$	**2)** $\Rightarrow x=\pm\sqrt{\dfrac{y+b}{a}}$ $y=ax^2-b$: • $ax^2=y+b$ • $x^2=\dfrac{y+b}{a}$ • $x=\pm\sqrt{\dfrac{y+b}{a}}$
3) $\Rightarrow x=\sqrt[3]{\dfrac{y+b}{a}}$ $y=ax^3-b$: • $ax^3=y+b$ • $x^3=\dfrac{y+b}{a}$ • $x=\sqrt[3]{\dfrac{y+b}{a}}$	**4)** $\Rightarrow x=1\pm\sqrt{y-2}$ $y=x^2-2x+3$: This is a quadratic equation. To solve for x, we can complete the square or use the quadratic formula: • $x^2-2x=y-3$ • $x^2-2x+1=y-3+1$ • $(x-1)^2=y-2$ • $x-1=\pm\sqrt{y-2}$ • $x=1\pm\sqrt{y-2}$
5) $\Rightarrow x=\dfrac{1}{(y+\frac{1}{2})^2-\frac{1}{4}}$ $y=x^{-1}-(x+1)^{-1}$: To solve for x, we need to find a common denominator and combine the terms: • $y=\dfrac{1}{x}-\dfrac{1}{x+1}$ • $y=\dfrac{(x+1)-x}{x(x+1)}$ • $y=\dfrac{1}{x(x+1)}=\dfrac{1}{x^2+x+\frac{1}{4}-\frac{1}{4}}=\dfrac{1}{(x+\frac{1}{2})^2-\frac{1}{4}}$	**6)** $\Rightarrow x=\dfrac{3y+1}{2y-1}$ To solve for x, we cross-multiply: • $y(2x-3)=x+1$ • $2yx-3y=x+1$ • $2yx-x=3y+1$ • $x(2y-1)=3y+1$ • $x=\dfrac{3y+1}{2y-1}$ As long as $2y-1\neq 0$, this is the **Solution** for x.

#1-03. Quadratic Functions: Completing the Square to Find the Vertex

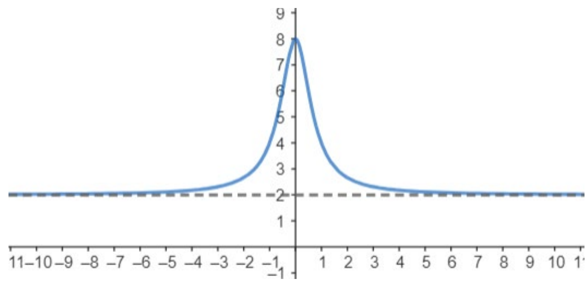
Using Completing square and find a vertex.

1) $y = x^2 - 4x + 4$	2) $y = x^2 - 4x$
3) $y = 2x^2 - 4x$	4) $y = -4x^2 - 8x + 3$
5) $y = 2x^2 - 3x - 4$	

- No oblique asymptote or holes.

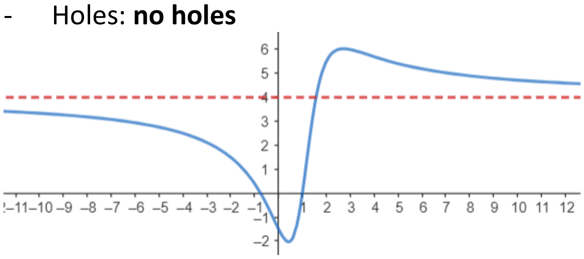


- Horizontal asymptote: $y=2$.
- No oblique asymptote or holes.



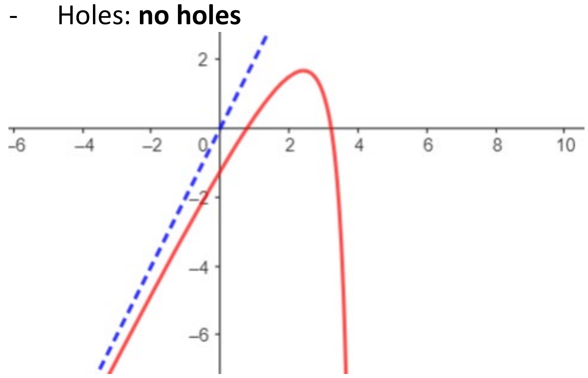
7) $f(x)=\dfrac{4x^2-x-3}{x^2-2x+2}$:

- Zeros: Set the numerator equal to zero and solve for x: $4x^2-x-3=0 \Rightarrow (4x+3)(x-1)=0$
  $x=-\dfrac{3}{4}$, 1
- Vertical Asymptotes: $x^2-2x+2=0 \Rightarrow$ No solutions, so no VAs.
- Horizontal Asymptote: $y=\dfrac{4}{1}=4$.
- Holes: **no holes**



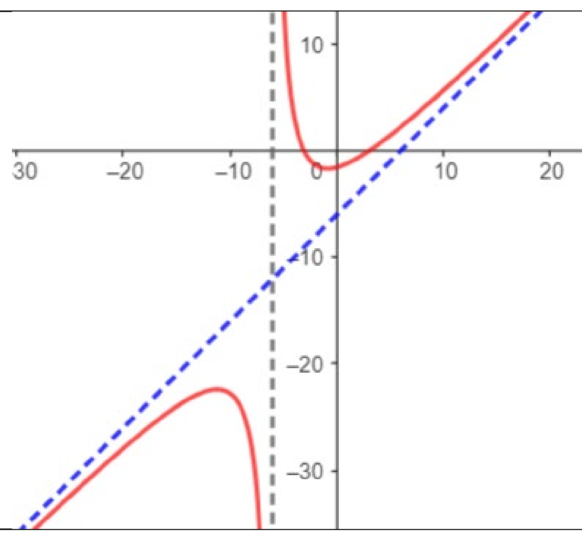
8) $f(x)=\dfrac{2x^2-8x+5}{x-4}=2x+\dfrac{5}{x-4}$

- Zeros: $2x^2-8x+5=0 \Rightarrow$
  $x=2+\dfrac{\sqrt{6}}{2}$ *and* $x=2-\dfrac{\sqrt{6}}{2}$
- Vertical Asymptotes: $x+4=0 \Rightarrow x=-4$.
- Horizontal Asymptote: no.
- Oblique Asymptote: $y=2x$
- Holes: **no holes**



9) $f(x)=\dfrac{x^2-9}{x+6}=x-6+\dfrac{27}{x+6}$:

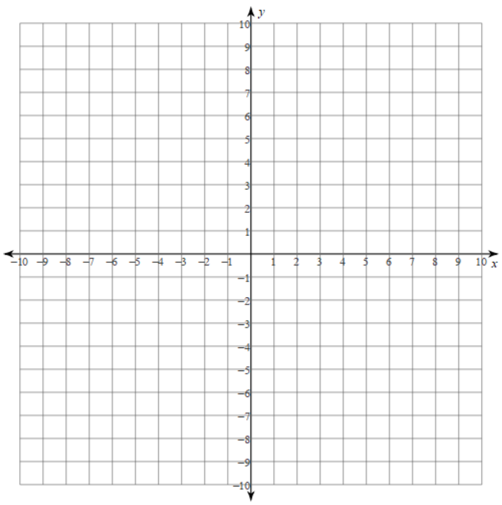
- Zeros: $x^2-9=0 \Rightarrow$ $x=3$ and $x=-3$
- Vertical Asymptote: $x+6=0 \Rightarrow x=-6$.
- Horizontal Asymptote: No($+\infty$ *or* $-\infty$).
- Oblique Asymptote: $y=x-6$.
- Holes: no holes.



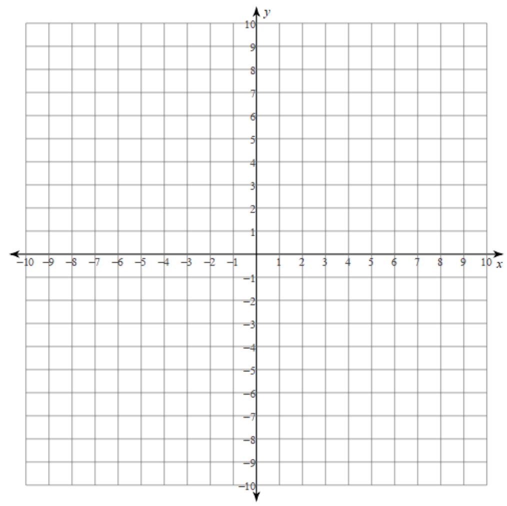
#1-29. Sketching Rational Functions: Asymptotes and Zeros

Draw graph briefly. Consider zeros, vertical asymptote, horizontal asymptote (end behavior), oblique asymptote and other constraint like holes.

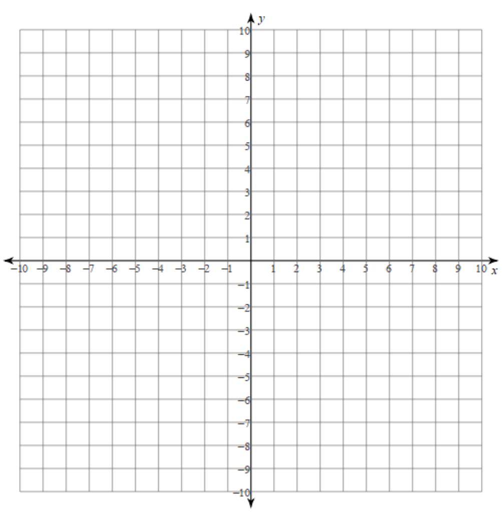
1) $f(x)=\dfrac{x^2-4}{x^2-16}$



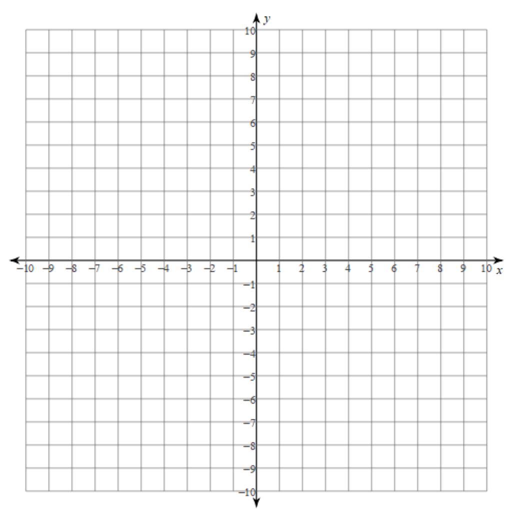
2) $f(x)=\dfrac{2x^3}{x^2-6x+5}$



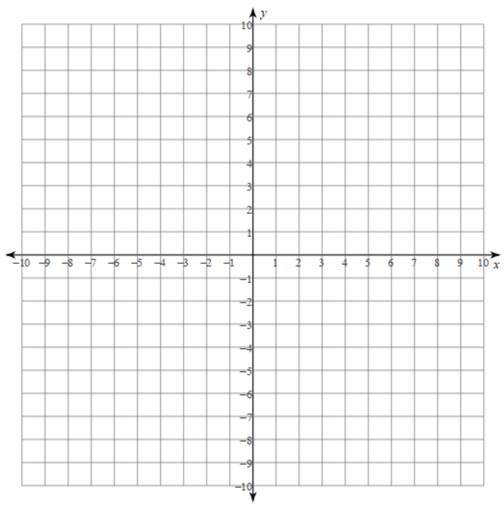
3) $f(x)=\dfrac{x^2-4}{x^2-2x-8}$



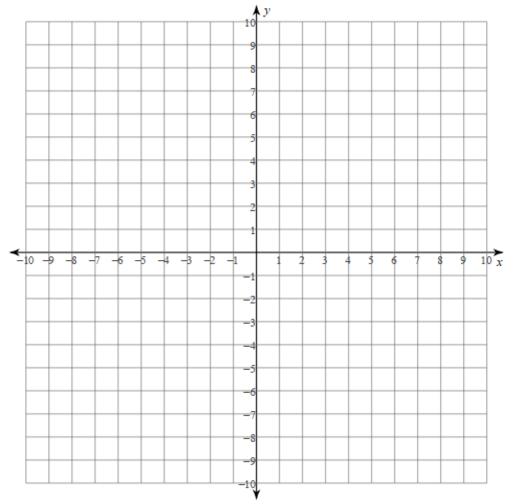
4) $f(x)=\dfrac{x}{x^2-2x-8}$



5) $f(x)=\dfrac{7}{x^2-2x-8}$



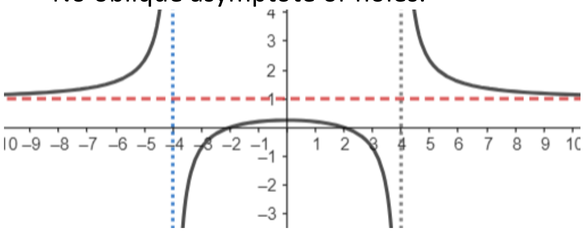
6) $f(x)=\dfrac{x^2-6x+5}{x-2}$



Solutions:

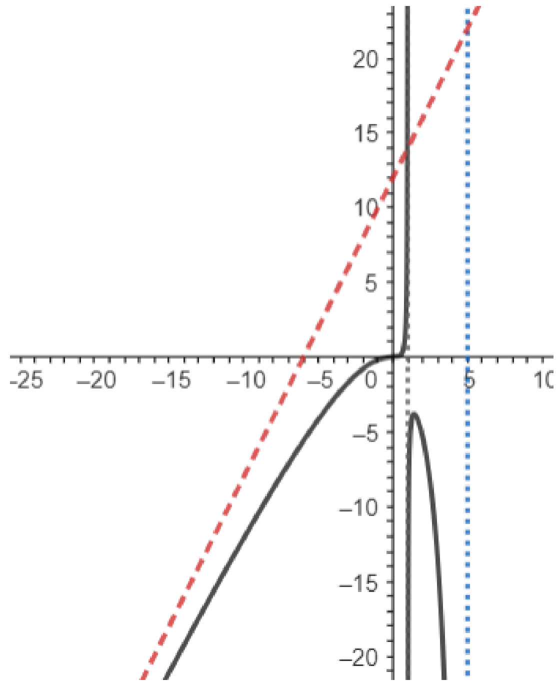
1) $f(x)=\dfrac{x^2-4}{x^2-16}$:

- Zeros: $x=-2$ and $x=2$.
- Vertical asymptotes: $x=-4$ and $x=4$.
- Horizontal asymptote: $y=1$.
- No oblique asymptote or holes.



2) $f(x)=\dfrac{2x^3}{x^2-6x+5}=2x+12+\dfrac{62x-60}{(x-5)(x-1)}$:

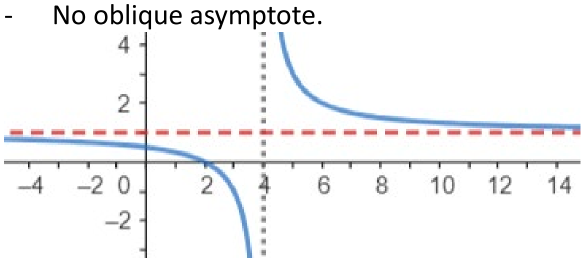
- Zeros: $x=0$.
- Vertical asymptotes: $x=1$ and $x=5$.
- No horizontal asymptote.
- Oblique asymptote: $y=2x+12$.
- No holes.



3) After cancellation:

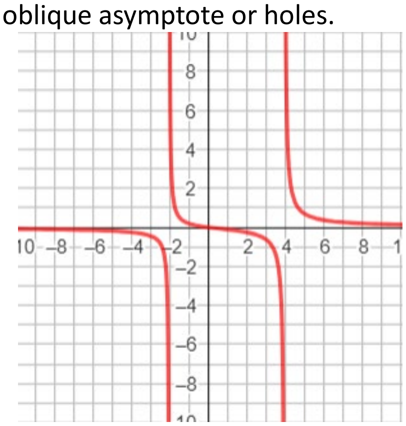
$$f(x)=\frac{x^2-4}{x^2-2x-8}=\frac{(x-2)(x+2)}{(x-4)(x+2)}=\frac{(x-2)}{(x-4)}$$

- Zeros: $x=2$.
- Vertical asymptotes: $x=4$. (Note: no VA at $x=-2$)
- Horizontal asymptote: $y=1$.
- Hole: $(-2,\frac{2}{3})$
- No oblique asymptote.



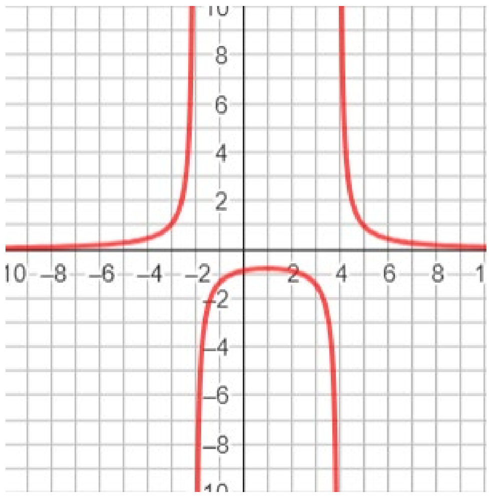
4) $f(x)=\dfrac{x}{x^2-2x-8}=\dfrac{x}{(x-4)(x+2)}$

- Zeros: $x=0$.
- Vertical asymptotes: $x=-2$ and $x=4$.
- Horizontal asymptote: $y=0$.
- No oblique asymptote or holes.



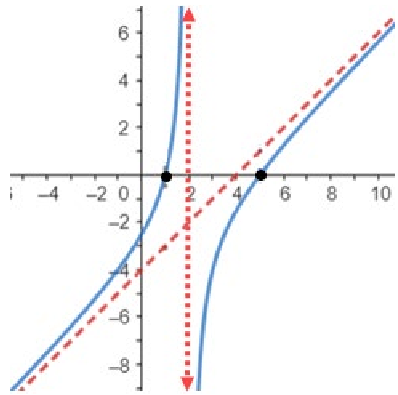
5) $f(x)=\dfrac{7}{x^2-2x-8}$:

- Zeros: None.
- Vertical asymptotes: $x=-2$ and $x=4$.
- Horizontal asymptote: $y=0$.
- No oblique asymptote or holes.



6) $f(x)=\dfrac{x^2-6x+5}{x-2}=x-4-\dfrac{13}{x-2}$

- Zeros: $x=1$ and $x=5$.
- Vertical asymptote: $x=2$.
- No horizontal asymptote.
- Oblique asymptote: $y=x-4$.
- No holes.



Chapter 2. Number Sense

#2-01. Number Sets: Counting and Categorizing Within a Range

Count the numbers within the specified range below.

$-11 \le x < 99$

1) Integers in the range	2) Whole numbers
3) Odd integers (including both positive & negative)	4) Even numbers (including both positive & negative)
5) Natural numbers	6) Multiples of 3 (including both positive & negative)
7) Negative Integers	

Solutions:

Count the numbers within the specified range below. $-11 \le x < 99$

1) $\Rightarrow$ Integers in the range: 110 - The integers from -11 to 98 (since $x < 99$) are counted as follows: $98-(-11)+1=110$.	**2)** $\Rightarrow$ Whole numbers: 99 - Whole numbers are non-negative integers. From 0 to 98 : $98-0+1=99$.
3) $\Rightarrow$ Odd integers (including both positive & negative): 55 - The first odd integer in the range is -11 and the last is 97. - There are $\frac{97-(-11)}{2}+1=55$ odd integers.	**4)** $\Rightarrow$ Even numbers (including both positive & negative): 55 - The first even integer in the range is -10 and the last is 98. - There are $\frac{98-(-10)}{2}+1=55$ even integers.
5) $\Rightarrow$ Natural numbers: 98 - Natural numbers are positive integers starting from 1. From 1 to 98: $98-1+1=98$.	**6)** $\Rightarrow$ Multiples of 3 (including both positive & negative): 36 - The first multiple of 3 in the range is -9 and the last is 99 (but since $x < 99$, we use 96). - There are $\frac{96-(-9)}{3}+1=36$ multiples of 3.
7) $\Rightarrow$ Negative Integers: 11 - The negative integers from -11 to -1: $-1-(-11)+1=11$.	

Solutions:

1) $\Rightarrow$ **a)** $t=1$ and $t=8$ seconds **b)** $t=9$

$h=144t-16t^2$ **.**

a) Find the number of seconds that the ball is in the air when it reaches a height of 128 feet.

- To find the time when the ball reaches 128 feet, we set $h=128$ and solve for t : $128=144t-16t^2 \Rightarrow$ $16t^2-144t+128=0 \Rightarrow t^2-9t+8=(t-8)(t-1)=0$
- The two possible times are $t=1$ seconds and $t=8$ seconds. Since both times are positive, the ball reaches a height of 128 feet twice, once on the way up and once on the way down.

b) After how many seconds does the ball hit the ground?

- The ball hits the ground when $h=0$. We use the same quadratic equation as before but look for the positive root: $0=-16t^2+144t=t(-16t+144) \Rightarrow$ $t=9$
- The ball hits the ground after 9 seconds.

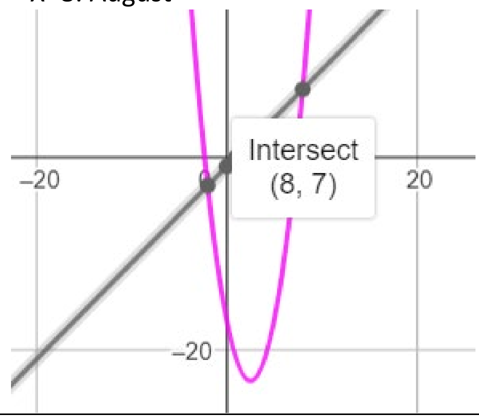
2) $\Rightarrow$ **x = 8 (August)**

$A(x)=x^2-5x-17$ **,** $B(x)=x-1$

- To find the prices at which both stocks had the same value, we set $A(x)=B(x)$ and solve for x :

  $x^2-5x-17=x-1 \Rightarrow$

  $x^2-6x-16=0 \Rightarrow (x-8)(x+2)=0$
- X=8: August



3) $\Rightarrow$ 7 inches

- Let w be the width of the metal sheet. Then the length is 3w. After cutting out squares of side length 2 inches from each corner and folding up the sides, the new dimensions of the box will be $w-4$ inches in width, $3w-4$ inches in length, and 2 inches in height.
- The volume V of the box is given by: $V=(w-4)(3w-4)(2) \Rightarrow 102=2(w-4)(3w-4) \Rightarrow$ $0=3w^2-16w-35$
- We can solve for w using the quadratic formula: $w=\dfrac{16\pm\sqrt{(-16)^2-4(3)(-35)}}{2(3)}=\dfrac{16\pm 26}{6}$
- We discard the negative solution since width cannot be negative: $w=\dfrac{16+26}{6}=7$
- The width of the rectangular sheet of metal should be 7 inches.

4) $\Rightarrow$ **Maximum area: 200 square yards**

- Let w be the width of the plot perpendicular to the river, and l be the length of the plot along the river. Since the river bounds one side, only three sides need fencing, so the total fencing used is $2w+l=40$.
- To maximize the area A , we use the formula $A=w\cdot l$. We can express l in terms of w using the fencing constraint: $l=40-2w$
- Now, the area in terms of w is $A(w)=w(40-2w)$ $=40w-2w^2$ $=-2(w^2-20w+100-100)$ $=-2(w-10)^2+200$
- Maximum area: at w=10, Area=200 square yards.

Examples (continued):

5) For two below questions, the profit that a coat manufacturer makes each day is modeled by the function $P(x) = -x^2 + 120x - 2000$ where $P(x)$ is the profit and x is the price of each coat sold.

a) For what values of x does the company make a profit?

b) What is the maximum profit that the company can make each day?

6) A rectangular sheet of aluminum 25 feet long and 12 inches wide is to be made into a rain gutter by folding up two parallel sides the same number of inches at right angles to the sheet. How many inches on each side should be folded up so that the gutter will have its greatest capacity?

7) A homeowner wants to increase the size of a rectangular deck that now measures 15 feet by 20 feet, but the building-code laws state that a homeowner cannot have a deck larger than 900 square feet. If the length and the width are to be increased by the same amount, find, to the nearest tenth, the maximum number of feet by which the length of the deck may legally be increased.

Solutions:

5) $\Rightarrow$ **a)** $20 < x < 100$ **b)** \$1600

$P(x) = -x^2 + 120x - 2000$

a) For what values of x does the company make a profit?

- The company makes a profit when $P(x) > 0$. To find the values of x for which this is true, we need to determine the roots of the quadratic equation $P(x) = 0$: $-x^2 + 120x - 2000 = 0$
- Let's use the quadratic formula to find the roots: $x = \dfrac{-120 \pm \sqrt{120^2 - 4(-1)(-2000)}}{2(-1)}$
- The two roots are: $x = \dfrac{-120 + 80}{-2} = 20$, $x = \dfrac{-120 - 80}{-2} = 100$
- The company makes a profit for $20 < x < 100$.

b) What is the maximum profit that the company can make each day?

- The maximum profit occurs at the vertex of the parabola. The vertex x value can be found using the formula $x = -\dfrac{b}{2a}$ for a quadratic function in the form $P(x) = ax^2 + bx + c$.
- For our function, $a = -1$ and $b = 120$, so: $x = -\dfrac{120}{2(-1)} = 60$
- Now we find the maximum profit by substituting $x = 60$ into the function: $P(60) = -(60)^2 + 120(60) - 2000 = 1600$
- The maximum profit that the company can make each day is \$1600.

6) $\Rightarrow$ 3 inches on each side

- Let x be the number of inches folded up on each side. The new width of the gutter will be $12 - 2x$ inches, and the height will be x inches. The length remains 25 feet (300 inches).
- The volume V of the gutter is given by: $V = (12 - 2x)x(300) = 3600x - 600x^2 = -600(x^2 - 6x + 9 - 9)$ $= -600(x-3)^2 + 5400$
- Maximum volume: at x=3, volume=5400
- The gutter will have its greatest capacity when 3 inches on each side are folded up.

7) $\Rightarrow$ maximum number of feet: 12.6 feet.

- Let x be the number of feet by which the length and the width are increased. The new dimensions of the deck will be $15 + x$ feet in width and $20 + x$ feet in length. The area A of the deck is: $A = (15 + x)(20 + x) = 300 + 35x + x^2$
- The maximum area allowed by law is 900 square feet, so we set $A \le 900$: $300 + 35x + x^2 \le 900 \Rightarrow$ $x^2 + 35x - 600 \le 0$ (→ solution continued next page)

- To find the maximum x , we solve the quadratic equation $x^2 + 35x - 600 = 0$ using the quadratic formula: $x = \dfrac{-35 \pm \sqrt{35^2 - 4(1)(-600)}}{2(1)} = \dfrac{-35 \pm \sqrt{3625}}{2} = \dfrac{-35 \pm 60.208}{2}$
- We take the positive root since x must be positive: $x = \dfrac{25.208}{2} = 12.604$
- To the nearest tenth, the maximum number of feet by which the length of the deck may legally be increased is 12.6 feet.

Chapter 3. Geometry and Trigonometry

#3-01. Trigonometric Functions and Unit Conversion Between Degrees and Radians

1. Fill in the blanks. Convert Radian to Degree, or Degree to Radian

Unit Conversion: $\pi = 180^o$ *or* $2\pi = 360^o$

(Radian)	0	π/6	(1)	π/3	π/2	(2)	3π/4	7π/6	(3)
(Degree)	0°	(4)	45°	(5)	(6)	180°	(7)	(8)	360°

2. Fill in the blanks without using calculator.

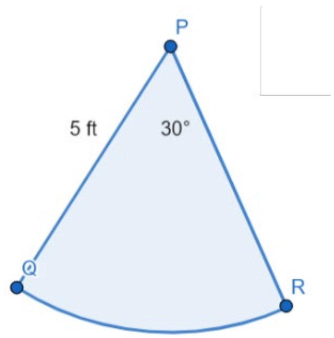
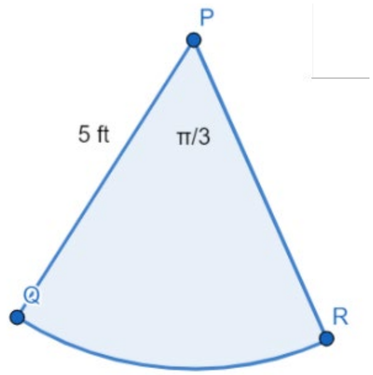
Θ (Degree)	**0°**	**30°**	**45°**	**60°**	**90°**	**180°**
sin Θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
cos Θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
tan Θ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	0
cosec Θ (csc Θ)	(1)	(2)	(3)	(4)	(5)	(6)
sec Θ	(7)	(8)	(9)	(10)	(11)	(12)
cot Θ	(13)	(14)	(15)	(16)	(17)	(18)

Solutions:

1. Fill in the blanks. Convert Radian to Degree, or Degree to Radian **1)** $\Rightarrow$ **π/4** **2)** $\Rightarrow$ **π** **3)** $\Rightarrow$ **2π** **4)** $\Rightarrow$ **30°** **5)** $\Rightarrow$ **60°** **6)** $\Rightarrow$ **90°** **7)** $\Rightarrow$ **135°** **8)** $\Rightarrow$ **210°**	2. Fill in the blanks without using calculator. **1)** $\Rightarrow \infty$ **2)** $\Rightarrow$ **2** **3)** $\Rightarrow \sqrt{2}$ **4)** $\Rightarrow \frac{2\sqrt{3}}{3}$ **5)** $\Rightarrow$ **1** **6)** $\Rightarrow \infty$ **7)** $\Rightarrow$ **1** **8)** $\Rightarrow \frac{2\sqrt{3}}{3}$ **9)** $\Rightarrow \sqrt{2}$ **10)** $\Rightarrow$ **2** **11)** $\Rightarrow \infty$ **12)** $\Rightarrow$ **-1** **13)** $\Rightarrow \infty$ **14)** $\Rightarrow \sqrt{3}$ **15)** $\Rightarrow$ **1** **16)** $\Rightarrow \frac{\sqrt{3}}{3}$ **17)** $\Rightarrow$ **0** **18)** $\Rightarrow -\infty$

#3-03. Geometry Problems: Arc Length, Sector Area, and Surface Area & Volume Calculations

1. Find arc length $\widehat{RQ}$ and area PQR.

1) Center angle 30° and r = 5 ft 	2) Center angle $\frac{\pi}{3}$ and r = 5 ft 

2. Find surface area and volume of the cone and cylinder.

1) Cone height = 10 ft and radius of bottom = 5 ft	2) Cone slant (side) height = 10 ft and radius of bottom = 5 ft
3) Cylinder height = 10 ft and radius of bottom = 5 ft	

Solutions:

1.

1) $L\left(\widehat{RQ}\right)=2\pi r\times\frac{30}{360}=\frac{5\pi}{6}\,(\text{ft})$, $A=\pi r^2\times\frac{30}{360}=\frac{25\pi}{12}(\text{ft}^2)$	**2)** $L\left(\widehat{RQ}\right)=r\theta=\frac{5\pi}{3}(\text{ft})$, $A=\frac{1}{2}r\theta^2=\frac{25\pi}{6}(\text{ft}^2)$

2. To find the surface area and volume of a cone and a cylinder, we use the following formulas:

For a cone:

- Volume: $V=\frac{1}{3}\pi r^2 h$
- Lateral surface area: $A_{\text{lateral}}=\pi rl$
- Total surface area: $A_{\text{total}}=\pi r(r+l)$

where r is the radius, h is the height, and l is the slant height of the cone.

For a cylinder:

- Volume: $V=\pi r^2 h$
- Lateral surface area: $A_{\text{lateral}}=2\pi rh$
- Total surface area: $A_{\text{total}}=2\pi rh+2\pi r^2$

where r is the radius and h is the height of the cylinder.

1) Cone with height $h=10$ ft and radius $r=5$ ft: The slant (side) height l can be found using the Pythagorean theorem: $l=\sqrt{r^2+h^2}=\sqrt{5^2+10^2}$ $=\sqrt{25+100}=\sqrt{125}=5\sqrt{5}$ ft. • Volume: $V=\frac{1}{3}\pi(5)^2(10)=\frac{1}{3}\pi\cdot 25\cdot 10$ $=\frac{250\pi}{3}$ ft^3 • Total surface area: $A_{\text{total}}=\pi(5)(5+5\sqrt{5})$ $=\pi(25+25\sqrt{5})=25\pi(1+\sqrt{5})$ ft^2	**2) Cone with lateral height $l=10$ ft and radius $r=5$ ft:** Here, the height of the cone is $5\sqrt{3}$. • Volume: $V=\frac{1}{3}\pi(5)^2(5\sqrt{3})=\frac{1}{3}\pi\cdot 25\cdot(5\sqrt{3})$ $=\frac{125\sqrt{3}\pi}{3}$ ft^3 • Total surface area: $A_{\text{total}}=\pi(5)(5+10)=\pi(25+50)=75\pi$ ft^2
3) **Cylinder with height $h=10$ ft and radius $r=5$ ft:** • Volume: $V=\pi(5)^2(10)=\pi\cdot 25\cdot 10=250\pi$ ft^3 • Total surface area: $A_{\text{total}}=2\pi(5)(10)+2\pi(5)^2=100\pi+50\pi$ $=150\pi$ ft^2 These are the surface areas and volumes for the cone and cylinder with the given dimensions.	

#3-05. Analytical Geometry: Angles and Distances in Coordinate Geometry

Solve.

1) Find a smaller angle (θ) between a line $x-\sqrt{3}y+4=0$ and x-axis	2) Find an angle (θ) between two lines $\begin{cases} x-y+1=0 \\ \sqrt{3}x-y-4=0 \end{cases}$
3) A shortest line length between a point $(3,1)$ and a line $2x+3y+4=0$	4) A shortest line length between two lines $\begin{cases} x+2y+1=0 \\ x+2y-4=0 \end{cases}$

Solutions:

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1) $\Rightarrow \theta = \arctan\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$

Angle θ between the line $x - \sqrt{3}y + 4 = 0$ and the x-axis:

First, find the slope of the line by rewriting the equation in slope-intercept form ($y = mx + b$):

$$x - \sqrt{3}y + 4 = 0$$
$$\sqrt{3}y = x + 4$$
$$y = \frac{1}{\sqrt{3}}x + \frac{4}{\sqrt{3}}$$

The slope m is $\frac{1}{\sqrt{3}}$. To find the angle θ between the line and the x-axis, use the arctan of the slope:

$\theta = \arctan\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$

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2) $\Rightarrow \theta = \arctan\left(\frac{\sqrt{3}-1}{1+\sqrt{3}}\right) = 15^\circ$

Angle θ **between the lines** $x - y + 1 = 0$ **and** $\sqrt{3}x - y - 4 = 0$**:**

Find the slopes of both lines:
For $x - y + 1 = 0$, the slope m_1 is 1 (after rearranging to $y = x + 1$).
For $\sqrt{3}x - y - 4 = 0$, the slope m_2 is $\sqrt{3}$ (after rearranging to $y = \sqrt{3}x - 4$).
The angle θ between the two lines can be found using the formula: $\theta = \arctan\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right)$

$\theta = \arctan\left(\frac{\sqrt{3}-1}{1+\sqrt{3}}\right) = 15^\circ$

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3) $\Rightarrow d = \sqrt{13}$

Distance between (3,1) and line $2x + 3y + 4 = 0$ **:**

The shortest distance from a point to a line is the length of the perpendicular from the point to the line. The formula for this distance is:

$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

For the point $(3,1)$ and the line $2x + 3y + 4 = 0$, where $A = 2$, $B = 3$, and $C = 4$, the distance d is:

$d = \frac{|2(3) + 3(1) + 4|}{\sqrt{2^2 + 3^2}}$

$d = \sqrt{13}$

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4) $\Rightarrow d = \frac{|-4-1|}{\sqrt{1^2 + 2^2}} = \sqrt{5}$

Distance between x + 2y + 1 = 0 and $x + 2y - 4 = 0$ **:**

Since the lines are parallel, the shortest distance is the perpendicular distance between them. The formula for the distance between parallel lines is:

$d = \frac{|C_2 - C_1|}{\sqrt{A^2 + B^2}}$

For the lines $x + 2y + 1 = 0$ and $x + 2y - 4 = 0$, where $A = 1$, $B = 2$, $C_1 = 1$, and $C_2 = -4$, the distance d is: $d = \frac{|-4-1|}{\sqrt{1^2 + 2^2}} = \sqrt{5}$

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#3-06. Solving Radical and Absolute Value Equations

Solve equations.

1) $\sqrt{x} = x$	2) $\sqrt{4-x} = \sqrt{x}$
3) $\sqrt{x+1} = x$	4) $\sqrt{x-1} + \sqrt{x-3} = 3$
5) $\sqrt{x-1} = x$	6) $\sqrt{x-1} + \sqrt{3-x} = 3$
7) $\sqrt{x+2} = x$	8) $\lvert x-1\rvert + \lvert x\rvert = 3$
9) $\sqrt{2-x} = x$	10) $\sqrt{x} = \lvert x\rvert$

Solutions:

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1) $\Rightarrow x=0$ and $x=1$

$\sqrt{x}=x$
Squaring both sides gives $x=x^2$. This simplifies to $x^2-x=0$, so $x(x-1)=0$. The **solutions** are $x=0$ and $x=1$. Both are valid since they satisfy the original equation.

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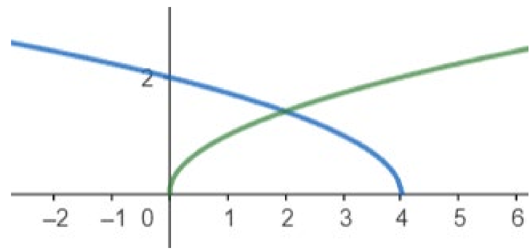
2) $\Rightarrow x=2$

$\sqrt{4-x}=\sqrt{x}$
Squaring both sides gives $4-x=x$. This simplifies to $4=2x$, so $x=2$. This is valid since it satisfies the original equation.



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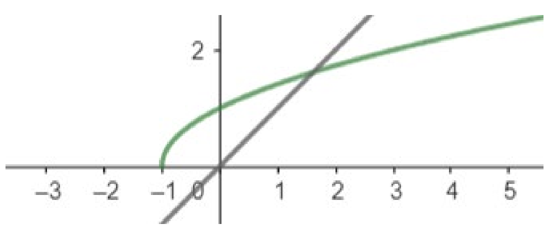
3) $\Rightarrow x=\dfrac{1+\sqrt{5}}{2}$

$\sqrt{x+1}=x$
Squaring both sides gives $x+1=x^2$. This simplifies to $x^2-x-1=0$. This quadratic equation does not factor easily, so we can use the quadratic formula to find $x=\dfrac{1\pm\sqrt{5}}{2}$. Only the positive **solution** $x=\dfrac{1+\sqrt{5}}{2}$ is valid since x must be greater than or equal to 0.



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4) $\Rightarrow x=\dfrac{157}{36}$

To solve the equation $\sqrt{x-1}+\sqrt{x-3}=3$, we will isolate one of the square roots and then square both sides to eliminate the square root. Here are the steps:
- Isolate one of the square roots: $\sqrt{x-1}=3-\sqrt{x-3}$
- Square both sides of the equation to eliminate the square root on the left side: $(\sqrt{x-1})^2=(3-\sqrt{x-3})^2$
- Simplify the right side of the equation: $x-1=9-6\sqrt{x-3}+x-3$
- Combine like terms: $x-1=x+6-6\sqrt{x-3}$
- Move all terms involving x to one side: $6\sqrt{x-3}=7$
- Isolate the square root: $\sqrt{x-3}=\dfrac{7}{6}$
- Square both sides again to eliminate the square root: $(\sqrt{x-3})^2=\left(\dfrac{7}{6}\right)^2$
- Solve for x: $x=\dfrac{49}{36}+3=\dfrac{157}{36}$
The **solution** to the equation is $x=\dfrac{157}{36}$.

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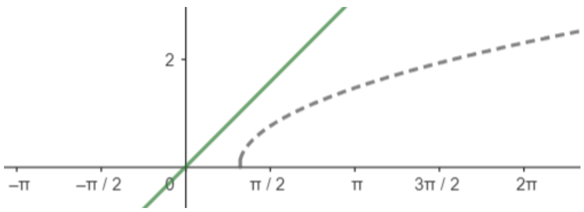
5) $\Rightarrow$ **no real solutions**

$\sqrt{x-1}=x$

Squaring both sides gives $x-1=x^2$. This simplifies to $x^2-x+1=0$. This quadratic equation has **no real solutions** since the discriminant is negative.

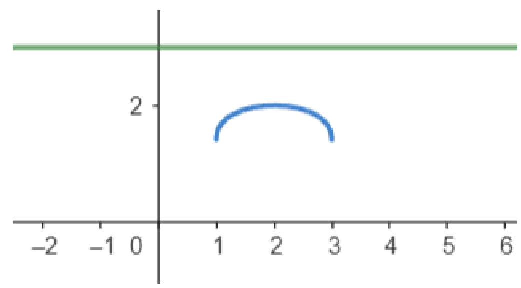


6) $\Rightarrow$ $\sqrt{x-1}+\sqrt{3-x}=3$ has **no solution.**

Here's a brief explanation:

- Isolate one square root: $\sqrt{x-1}=3-\sqrt{3-x}$.
- Square both sides: $x-1=9-6\sqrt{3-x}+3-x$.
- Simplify and isolate the square root: $6\sqrt{3-x}=12-x$.
- Square again to eliminate the square root: $36(3-x)=(12-x)^2$.
- Expand and simplify to form a quadratic equation: $x^2-10x-61=0$.
- Solve the quadratic equation: $x=5\pm\sqrt{86}$.

Since the domain of the original equation requires $1<x<3$, and neither $5-\sqrt{86}$ nor $5+\sqrt{86}$ falls within this interval, there is **no solution to the equation**.



7) $\Rightarrow$ $x=2$

To solve the equation $\sqrt{x+2}=x$, we will follow these steps:

- Note that the square root is only defined for non-negative numbers, so $x+2\geq 0$ which implies $x\geq -2$.
- Additionally, since the square root is on one side of the equation and x is on the other, x must also be non-negative (since a square root cannot be negative). Therefore, $x\geq 0$.
- Now, square both sides to eliminate the square root: $(\sqrt{x+2})^2=(x)^2$
- Simplify both sides: $x+2=x^2$
- Rearrange the terms to form a quadratic equation: $x^2-x-2=0$
- Factor the quadratic equation: $(x+1)(x-2)=0$
- Solve for x: $x=-1$ or $x=2$

8) $\Rightarrow$ $x=2$ and $x=-1$.

To solve the equation $|x-1|+|x|=3$, we need to consider different cases based on the value of x:

Case 1: $x\geq 1$

Both absolute values are non-negative, so the equation becomes: $(x-1)+x=3$

$x=2$. This **solution** is valid for Case 1 since $x\geq 1$.

Case 2: $0\leq x<1$

The first absolute value is non-negative, and the second is negative, so the equation becomes:

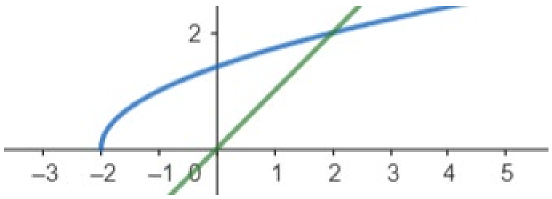
$(x-1)-x=3$

$-1=3$. This is not possible, so there are no **solutions** in this case.

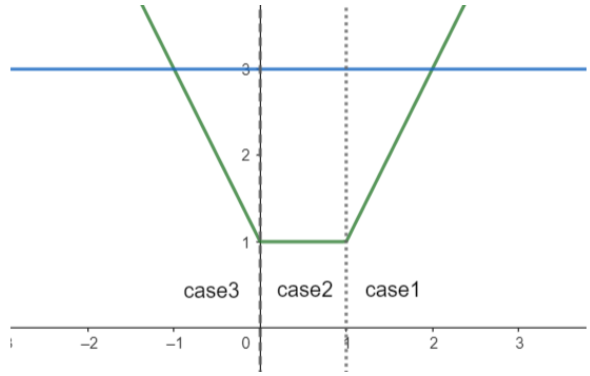
Case 3: $x<0$

Both absolute values are negative, so the equation becomes: $-(x-1)-x=3$

Now we need to check these **solutions** to ensure they satisfy the original equation. Only $x = 2$ satisfy the condition.



$x = -1$. This **solution** is valid for Case 3 since $x < 0$.
Therefore, the **solutions** to the equation are $x = 2$ and $x = -1$.



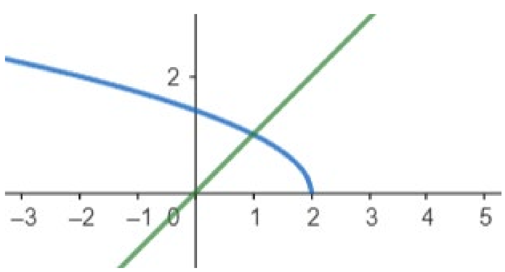
9) $\Rightarrow x = 1$

To solve the equation $\sqrt{2-x} = x$, we proceed as follows:

First, note that the square root requires $2 - x \geq 0$, which implies $x \leq 2$. Also, the square root is non-negative, so $x \geq 0$.

Squaring both sides gives us:
$2 - x = x^2 \Rightarrow x^2 + x - 2 = 0$
So, $x = -2$ or $x = 1$.

However, $x = -2$ **does not satisfy** the original equation because it would result in taking the square root of a negative number. Therefore, the **only solution is** $x = 1$**,** which is valid as it satisfies both the original equation and the condition $x \leq 2$.



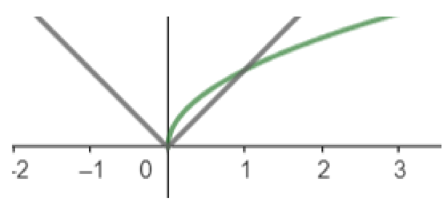
10) $\Rightarrow x = 0$, $x = 1$

To solve the equation $\sqrt{x} = |x|$, we consider two cases based on the value of x**:**

Case 1: $x \geq 0$
The equation becomes $\sqrt{x} = x$, which we can solve by squaring both sides: $x = x^2$
$x^2 - x = 0$
$x(x-1) = 0$
The **solutions** are $x = 0$ and $x = 1$, both of which are valid since they satisfy the original equation.

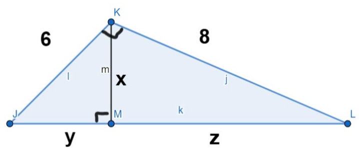
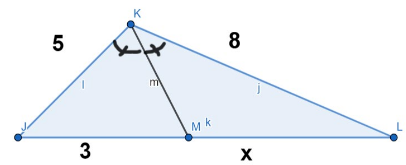
Case 2: $x < 0$
The equation becomes $\sqrt{x} = -x$, which has **no solution** because the square root of a negative number is not real, and the square root is non-negative while $-x$ is positive.

Therefore, the **solutions** to the equation are $x = 0$ and $x = 1$.

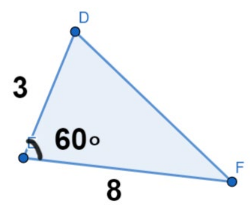
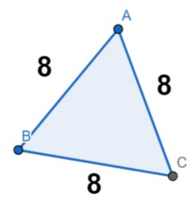
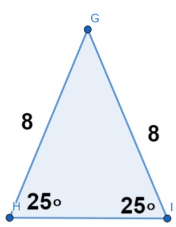
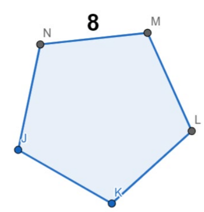
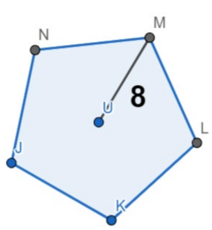
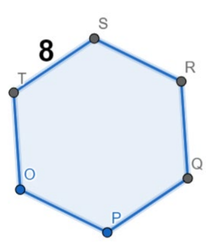
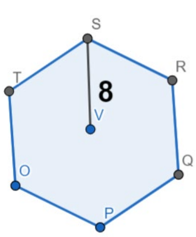


#3-07. Geometry Practice: Calculating Lengths and Areas

1. Find the length below. (These drawings are not properly portioned.)

1) Find x, y, and z 	2) Find x 

2. Find the area below. (These drawings are not properly portioned.)

1) Area for a triangle 	2) Area for an equilateral triangle 
3) Area for an isosceles triangle 	4) Area for a regular pentagon 
5) Area for a regular pentagon 	6) Area for a regular hexagon 
7) Area for a regular hexagon 	8) Find area of the triangle ABC is if the vertices are: A (1, 1), B (4, 5), and C (9, −5). (Hint: Use a formula by matrix determinant if you know)

Solutions:

1.

1) $\Rightarrow$ x=4.8, y=3.6, z=6.4 From the right triangle similarity, - $x^2 = y \cdot z$, - $6^2 = y \cdot (y+z)$, - $8^2 = z \cdot (y+z)$, - $(y+z) = 10$ from right triangle **Solutions**: x=4.8, y=3.6, z=6.4	**2)** $\Rightarrow$ x=$\frac{24}{5}$ From triangle similarity $\frac{5}{3} = \frac{8}{x}$ **Solution**: x=$\frac{24}{5}$

2.

1) $\Rightarrow$ $A = \frac{1}{2} \times 3 \times 4 \times \sin(60^\circ) = 6\sqrt{3}$	**2)** $\Rightarrow$ $A = \frac{1}{2} \times 8 \times 8 \times \sin(60^\circ) = 16\sqrt{3}$
3) $\Rightarrow$ $A = \frac{1}{2} \times 8 \times 8 \times \sin(130^\circ) = 24.51$	**4)** $\Rightarrow$ $A = \frac{1}{2} \times r \times 4 \times \sin(54^\circ) \times 10$ $= \frac{1}{2} \times (\frac{4}{\cos(54^\circ)}) \times 4 \times \sin(54^\circ) \times 10 = 110.1$
5) $\Rightarrow$ $A = \frac{1}{2} \times 8 \times 8 \times \sin(72^\circ) \times 5 = 152.2$	**6)** $\Rightarrow$ $A = \frac{1}{2} \times 8 \times 8 \times \sin(60^\circ) \times 6 = 96\sqrt{3}$
7) $\Rightarrow$ $A = \frac{1}{2} \times 8 \times 8 \times \sin(60^\circ) \times 6 = 96\sqrt{3}$	**8)** $\Rightarrow$ $A = \pm\frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 4 & 5 & 1 \\ 9 & -5 & 1 \end{vmatrix} = 25$ (*positive*)

#3-09. Range Operations Practice: Determining the Range of Variable Expressions

To find the range (minimum and maximum values) for each of these expressions using range operations

1) $1 < x < 2$ and $3 < y < 4$ a) $x + y$ b) $x - y$ c) $x \times y$ d) $x \div y$	2) $1 < x \le 2$ and $3 \le y < 4$ a) $x + y$ b) $x - y$ c) $x \times y$ d) $x \div y$
3) $-1 < x < 2$ and $-3 < y < 4$ a) $x + y$ b) $x - y$ c) $x \times y$ d) $x \div y$	4) $-2 < x < -1$ and $-4 < y < -3$ a) $x + y$ b) $x - y$ c) $x \times y$ d) $x \div y$

Solutions:

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1) To find the range for the expressions with $1<x<2$ and $3<y<4$, we'll perform range operations for each expression:

a) $x+y$:

- Minimum: When x is at its minimum and y is at its minimum, $1+3=4$.
- Maximum: When x is at its maximum and y is at its maximum, $2+4=6$.
- **Range:** $4<x+y<6$.

b) $x-y$:

- Minimum: When x is at its minimum and y is at its maximum, $1-4=-3$.
- Maximum: When x is at its maximum and y is at its minimum, $2-3=-1$.
- **Range:** $-3<x-y<-1$.

c) $x\times y$:

- Minimum: When x is at its minimum and y is at its minimum, $1\times3=3$.
- Maximum: When x is at its maximum and y is at its maximum, $2\times4=8$.
- **Range:** $3<x\times y<8$.

d) $x\div y$:

- Minimum: When x is at its minimum and y is at its maximum, $\frac{1}{4}$.
- Maximum: When x is at its maximum and y is at its minimum, $\frac{2}{3}$.
- **Range:** $\frac{1}{4}<\frac{x}{y}<\frac{2}{3}$.

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2) The ranges for $1<x\le2$ and $3\le y<4$ are:

a) $x+y$:

- Minimum: When x is at its minimum and y is at its minimum, $1+3=4$.
- Maximum: When x is at its maximum and y is at its maximum, $2+4=6$.
- **Range:** $4<x+y<6$.

b) $x-y$:

- Minimum: $1-4=-3$
- Maximum: $2-3=-1$
- **Range:** $-3<x-y<-1$

c) $x\times y$:

- Minimum: $1\times3=3$
- Maximum: $2\times4=8$
- **Range:** $3<x\times y<8$

d) $x\div y$:

- Minimum: $\frac{1}{4}$
- Maximum: $\frac{2}{3}$
- **Range:** $\frac{1}{4}<x\div y<\frac{2}{3}$

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3) To find the range for the expression $-1<x<2$ and $-3<y<4$ by performing range operations, we analyze each operation separately:

a) $-4<x+y<6$

b) $-5<x-y<5$

c) $-6<xy<8$

d) $-\infty<\frac{x}{y}<\infty$ (Minimum: $\frac{-1}{0}\cong-\infty$ and Maximum: $\frac{1}{0}\cong\infty$)

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4) For the range $-2<x<-1$ and $2<y<4$, here are the minimum and maximum values for each expression:

a) $-6<x+y<-4$

b) $1<x-y<3$

c) $3<xy<8$

d) $\frac{1}{4}<\frac{x}{y}<\frac{2}{3}$

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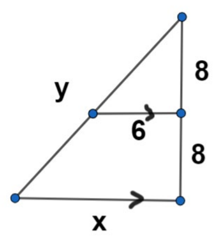
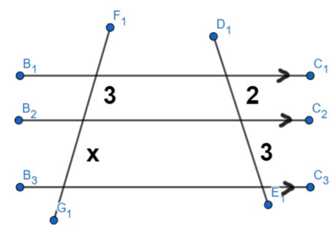
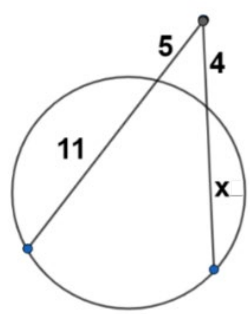
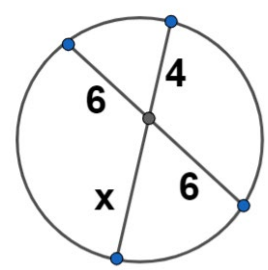
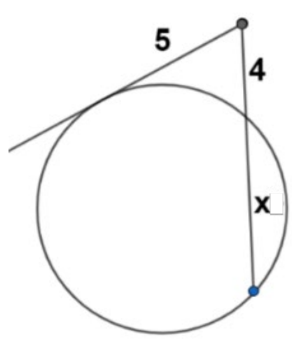
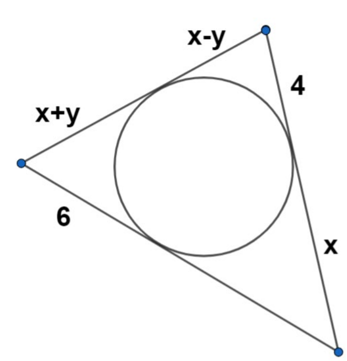
#3-10. Geometry Practice: Finding Lengths in Triangles and Circles

1. Find lengths by using triangle similarity. (These drawings are not properly portioned.)

1) Find x and y 	2) Find x where $c_1 \parallel c_2 \parallel c_3$ 

2. Find length x, y, and z in circles. (These drawings are not properly portioned.)

1) Find x 	2) Find x 
3) Find x 	4) Find x and y 

Solutions:

1.

1) $\Rightarrow$ x=12, y=20	**2)** $\Rightarrow \frac{3}{x}=\frac{2}{3} \Rightarrow x=\frac{9}{2}$

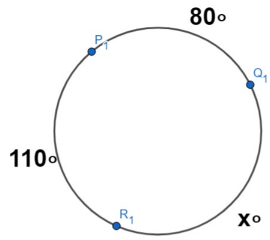
2.

1) $\Rightarrow 5\times(5+11)=4\times(4+x) \Rightarrow$ x=16	**2)** $\Rightarrow 6\times(6)=4\times(x) \Rightarrow$ x=9
3) $\Rightarrow 5^2=4(4+x) \Rightarrow x=\frac{9}{4}$	**4)** $\Rightarrow \begin{cases} x-y=4 \\ x+y=6 \end{cases} \Rightarrow$ x=5, y=1

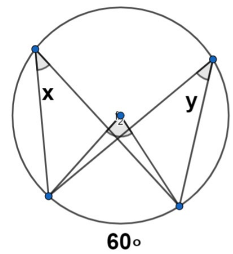
#3-11. Circle Geometry: Finding Missing Angles

1. **Find angles x, y, and z in circles.** (These drawings are not properly portioned.)

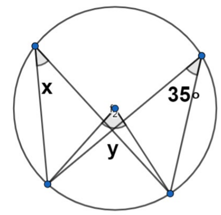
1) Find x



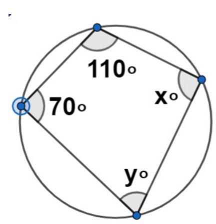
2) Find x and y



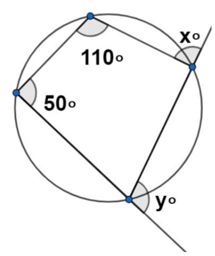
3) Find x and y



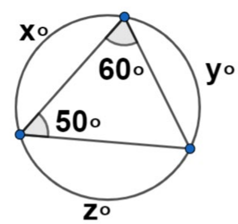
4) Find x and y



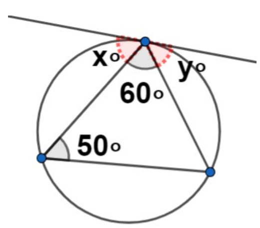
5) Find x and y



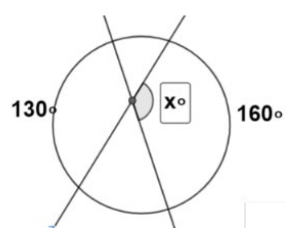
6) Find x, y and z



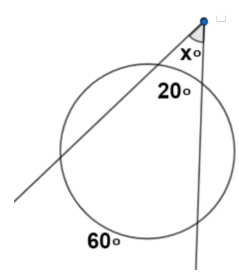
7) Find x and y



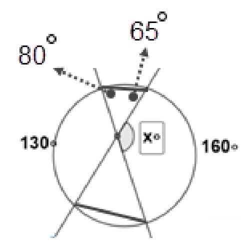
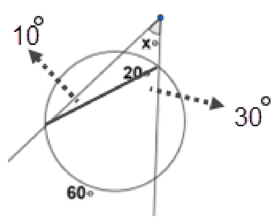
8) Find x



9) Find x



Solutions:

1) $\Rightarrow$ x=170° x = 360 – (110 +80) = 170	**2)** $\Rightarrow$ x=30°, y=30° $x = \frac{60}{2}$, $y = \frac{60}{2}$
3) $\Rightarrow$ x=35°, y=70° $y = 35 \times 2 = 70$, $x = 35$	**4)** $70^\circ + x = 180^\circ \Rightarrow x = 110^\circ$, $110^\circ + y = 180^\circ \Rightarrow y = 70^\circ$
5) $\Rightarrow$ x=50°, y=110° $50^\circ + (180 - x) = 180^\circ \Rightarrow x = 50^\circ$ $110^\circ + (180 - y) = 180^\circ \Rightarrow y = 110^\circ$	**6)** $\Rightarrow$ x=140°, y=100°, z= 120° $z = 2 \times 60 = 120$ $y = 2 \times 50 = 100$ x = 360 – (120 +100) = 140
7) $\Rightarrow$ x=70°, y=50°	**8)** $\Rightarrow$ x=145°  $x = 65 + 80 = 145$
9) $\Rightarrow$ x=20°  $x = 30 - 10 = 20$	

#3-12. Trigonometric Identities and Angle Relationships

Solve

1) If $\sin(90^o - \theta) = \frac{\sqrt{2}}{2}$ *where* $0^o < \theta < 90^o$, what is $\cos(\theta)$?	2) If $\sin(\theta) = \frac{1}{2}$ *where* $0^o < \theta < 90^o$, what is $\cos(90^o - \theta)$?
3) If $\sin(\theta) = \frac{1}{2}$ *where* $0^o < \theta < 90^o$, what is $\cos(\theta)$?	4) If $\sin(\theta) = \frac{\sqrt{2}}{3}$ *where* $0^o < \theta < 90^o$, what is $\cos(\theta)$?
5) If $\sin(\theta) = \frac{\sqrt{2}}{3}$ *where* $0^o < \theta < 90^o$, what is $\tan(\theta)$?	6) If $\cot(90^o - \theta) = \sqrt{3}$ *where* $0^o < \theta < 90^o$, what is $\tan(\theta)$?

Solutions:

1) $\Rightarrow \cos(\theta)=\frac{\sqrt{2}}{2}$ $\sin(90^{\circ}-\theta)=\frac{\sqrt{2}}{2}$ **where** $0^{\circ}<\theta<90^{\circ}$**:** - The identity $\sin(90^{\circ}-\theta)=\cos(\theta)$ applies here. - Therefore, $\cos(\theta)=\frac{\sqrt{2}}{2}$.	**2)** $\Rightarrow \cos(90^{\circ}-\theta)=\frac{1}{2}$ $\sin(\theta)=\frac{1}{2}$ **where** $0^{\circ}<\theta<90^{\circ}$**:** - Using the identity $\cos(90^{\circ}-\theta)=\sin(\theta)$, we find that $\cos(90^{\circ}-\theta)=\frac{1}{2}$.
3) $\Rightarrow \cos(\theta)=\frac{\sqrt{3}}{2}$ $\sin(\theta)=\frac{1}{2}$ **where** $0^{\circ}<\theta<90^{\circ}$**:** - To find $\cos(\theta)$, we use the Pythagorean identity $\sin^2(\theta)+\cos^2(\theta)=1$. - Substituting $\sin(\theta)=\frac{1}{2}$, we get $\left(\frac{1}{2}\right)^2+\cos^2(\theta)=1$. - Solving for $\cos(\theta)$ gives $\cos(\theta)=\frac{\sqrt{3}}{2}$.	**4)** $\Rightarrow \cos(\theta)=\frac{\sqrt{7}}{3}$ $\sin(\theta)=\frac{\sqrt{2}}{3}$ **where** $0^{\circ}<\theta<90^{\circ}$**:** - Using the Pythagorean identity, we substitute $\sin(\theta)=\frac{\sqrt{2}}{3}$ into $\sin^2(\theta)+\cos^2(\theta)=1$. - Solving for $\cos(\theta)$ gives $\cos(\theta)=\frac{\sqrt{7}}{3}$.
5) $\Rightarrow \tan(\theta)=\frac{\sin(\theta)}{\cos(\theta)}=\frac{\sqrt{14}}{7}$ $\sin(\theta)=\frac{\sqrt{2}}{3}$ **where** $0^{\circ}<\theta<90^{\circ}$**:** - The tangent of an angle is the ratio of the sine to the cosine. - Using the result from $\cos(\theta)=\frac{\sqrt{7}}{3}$, $\tan(\theta)=\frac{\sin(\theta)}{\cos(\theta)}=\frac{\frac{\sqrt{2}}{3}}{\frac{\sqrt{7}}{3}}=\frac{\sqrt{2}}{\sqrt{7}}=\frac{\sqrt{14}}{7}$.	**6)** $\Rightarrow \tan(\theta)=\sqrt{3}$ $\cot(90^{\circ}-\theta)=\sqrt{3}$ **where** $0^{\circ}<\theta<90^{\circ}$**:** - The identity $\cot(90^{\circ}-\theta)=\tan(\theta)$ applies here. - Therefore, $\tan(\theta)=\sqrt{3}$.

#3-13. Exploring Trigonometric Function Values and Angle Sum Identities

Solve

1) $\sin(75^\circ) = \sin(45^\circ + 30^\circ) = ?$	2) $\cos(15^\circ) = \sin(60^\circ - 45^\circ) = ?$
3) $\tan(75^\circ) = \tan(45^\circ + 30^\circ) = ?$	4) $\tan(15^\circ) = \tan(60^\circ - 45^\circ) = ?$
5) If $\sin(\theta - 90^o) = \frac{1}{4}$, what is $\cos(2\theta)$? $(0 < \theta < \pi)$	6) If $\sin(-\theta) = \frac{1}{4}$, what is $\sin(2\theta)$? $(\frac{\pi}{2} < \theta < \frac{3\pi}{2})$
7) If $\cos(-\theta) = \frac{\sqrt{2}}{3}$, what is $\cos(3\theta)$? (hint: use triple angle Identity formula)	

Solutions:

1) Using the sine addition formula:
$\sin(A+B)=\sin(A)\cos(B)+\cos(A)\sin(B)$, we get:

$$\sin(45^\circ+30^\circ)=\sin(45^\circ)\cos(30^\circ)+\cos(45^\circ)\sin(30^\circ)$$
$$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}=\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}$$

2) Using the sine subtraction formula:
$\sin(A-B)=\sin(A)\cos(B)-\cos(A)\sin(B)$, we get:

$$\cos(15^\circ)=\cos(60^\circ-45^\circ)$$
$$=\sin(60^\circ)\cos(45^\circ)-\cos(60^\circ)\sin(45^\circ)$$
$$=\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$$

3) Using the tangent addition formula:
$\tan(A+B)=\dfrac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}$, we get:

$$\tan(75^\circ)=\frac{\tan(45^\circ)+\tan(30^\circ)}{1-\tan(45^\circ)\tan(30^\circ)}=\frac{1+\frac{\sqrt{3}}{3}}{1-1\cdot\frac{\sqrt{3}}{3}}$$
$$=2+\sqrt{3}$$

4) Using the tangent subtraction formula:
$\tan(A-B)=\dfrac{\tan(A)-\tan(B)}{1+\tan(A)\tan(B)}$, we get:

$$\tan(15^\circ)=\frac{\tan(60^\circ)-\tan(45^\circ)}{1+\tan(60^\circ)\tan(45^\circ)}$$
$$=\frac{\sqrt{3}-1}{1+\sqrt{3}\cdot 1}=2-\sqrt{3}$$

5) Using the identity $\sin(\theta-90^\circ)=-\cos(\theta)$, we have $-\cos(\theta)=\frac{1}{4}$, so $\cos(\theta)=-\frac{1}{4}$. Now, using the double-angle formula for cosine, $\cos(2\theta)=2\cos^2(\theta)-1$, we get:

$$\cos(2\theta)=2\left(-\frac{1}{4}\right)^2-1=-\frac{7}{8}$$

6) Since $\sin(-\theta)=-\sin(\theta)$, we have $-\sin(\theta)=\frac{1}{4}\Rightarrow\sin(\theta)=-\frac{1}{4}$.

- Using the double-angle formula for sine, $\sin(2\theta)=2\sin(\theta)\cos(\theta)$, but we need $\cos(\theta)$ to proceed.
- Since $\sin^2(\theta)+\cos^2(\theta)=1$, we have: $\left(-\frac{1}{4}\right)^2+\cos^2(\theta)=1\Rightarrow\cos^2(\theta)=1-\frac{1}{16}=\frac{15}{16}$
- Since $\frac{\pi}{2}<\theta<\frac{3\pi}{2}$, $\cos(\theta)$ is negative, so $\cos(\theta)=-\frac{\sqrt{15}}{4}$.
- Now we can find $\sin(2\theta)$: $\sin(2\theta)=2\left(-\frac{1}{4}\right)\left(-\frac{\sqrt{15}}{4}\right)=\frac{\sqrt{15}}{8}$

7) Since $\cos(-\theta)=\cos(\theta)$, we have $\cos(\theta)=\frac{\sqrt{2}}{3}$.

- Using the triple-angle formula for cosine, $\cos(3\theta)=4\cos^3(\theta)-3\cos(\theta)$, we get:

$$\cos(3\theta)=4\left(\frac{\sqrt{2}}{3}\right)^3-3\left(\frac{\sqrt{2}}{3}\right)-\frac{19\sqrt{2}}{27}=\frac{-38\sqrt{2}}{27}$$

#3-14. Trigonometric Half-Angle Identities Practice

Find value by Trigonometry Half-Angle Identity

1) $\sin(22.5^o) = \sin(\frac{45^o}{2}) = ?$	2) $\tan(22.5^o) = \tan(\frac{45^o}{2}) = ?$
3) $\cos(15^o) = \sin(\frac{30^o}{2}) = ?$	

Solutions:

1) $\Rightarrow \sin\left(\frac{45^\circ}{2}\right)=\sqrt{\frac{1}{2}-\frac{\sqrt{2}}{4}}$ **To find** $\sin(22.5^\circ)=\sin\left(\frac{45^\circ}{2}\right)$**, we can use the half-angle formula for sine:** $\sin\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1-\cos(\theta)}{2}}$ For $\theta=45^\circ$, we have: $\sin\left(\frac{45^\circ}{2}\right)=\sqrt{\frac{1-\cos(45^\circ)}{2}}=\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}=\frac{\sqrt{2-\sqrt{2}}}{2}$ Since 22.5° is in the first quadrant, where sine is positive, we take the positive square root.	**2)** $\Rightarrow \tan\left(\frac{45^\circ}{2}\right)=\sqrt{2}-1$ **To find** $\tan(22.5^\circ)=\tan\left(\frac{45^\circ}{2}\right)$**, we can use the half-angle formula for tangent:** $\tan\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}}$ For $\theta=45^\circ$, we have: $\tan\left(\frac{45^\circ}{2}\right)=\sqrt{\frac{1-\cos(45^\circ)}{1+\cos(45^\circ)}}$ $=\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}}=\sqrt{3-2\sqrt{2}}=\sqrt{2}-1$ Since 22.5° is in the first quadrant, where tangent is positive, we take the positive square root.
3) $\Rightarrow \sin\left(\frac{30^\circ}{2}\right)=\frac{\sqrt{6}-\sqrt{2}}{4}$ **To find** $\cos(15^\circ)=\sin\left(\frac{30^\circ}{2}\right)$**, we can use the half-angle formula for sine:** $\sin\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1-\cos(\theta)}{2}}$ For $\theta=30^\circ$, we have: $\sin\left(\frac{30^\circ}{2}\right)=\sqrt{\frac{1-\cos(30^\circ)}{2}}=\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}=$ $=\frac{\sqrt{2-\sqrt{3}}}{2}=\frac{\sqrt{6}-\sqrt{2}}{4}$ Since 15° is in the first quadrant, where cosine is positive, we take the positive square root.	

#3-15. Complex Numbers and Polar Coordinates Conversion Exercises

Solve below where θ: [0, 2π)

1) Convert $(1+i)$ to polar coordinates.	2) Convert $(3i)$ to polar coordinates.
3) Convert $(1+i)^6$ to polar coordinates.	4) Convert $4\,cis(60^o)$ to rectangular coordinates.
5) Simplify $\left[2\,cis(30^o)\right]^4 \times 4\,cis(-60^o)$.	6) Write the four 4th roots of $16\,cis(120^o)$
7) Write the three cube roots of $(-i)$	8) Write the four 4th roots of $(1-\sqrt{3}i)$

Solutions:

1) In polar coordinates, a complex number $z=a+bi$ is represented as $z=r(\cos\theta+i\sin\theta)$, where r is the magnitude and θ is the angle. For $1+i$: - Magnitude: $r=\sqrt{1^2+1^2}=\sqrt{2}$ - Angle: $\theta=\arctan\left(\frac{1}{1}\right)=\arctan(1)=45^\circ$ or $\frac{\pi}{4}$ radians **Polar coordinates:** $\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$	**2)** For 3i (which is along the imaginary axis): - Magnitude: $r=\sqrt{0^2+3^2}=3$ - Angle: $\theta=\arctan\left(\frac{3}{0}\right)$ which is undefined, but since it's on the positive imaginary axis, $\theta=90^\circ$ or $\frac{\pi}{2}$ radians **Polar coordinates:** $3(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2})$
4) First, convert $1+i$ to polar coordinates as in (1), then raise it to the 6th power: - Magnitude: $r^6=(\sqrt{2})^6=8$ - Angle: $6\times45^\circ=270^\circ$ or $6\times\frac{\pi}{4}=\frac{3\pi}{2}$ radians **Polar coordinates:** $8(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2})$	**4)** Using $z=r(\cos\theta+i\sin\theta)$: - $z=4(\cos60^\circ+i\sin60^\circ)$ $=4\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=z=2+2\sqrt{3}i$ Rectangular coordinates: $2+2\sqrt{3}i$
5) First, raise $2\text{cis}(30^\circ)$ to the 4th power: - Magnitude: $2^4=16$ - Angle: $4\times30^\circ=120^\circ$ Now, multiply by $4\text{cis}(-60^\circ)$: - Magnitude: $16\times4=64$ - Angle: $120^\circ-60^\circ=60^\circ$ Result: $64\text{cis}(60^\circ)$	**6)** The magnitude of each root is the 4th root of 16, which is 2. The angles are $\frac{120^\circ+360^\circ k}{4}$ for $k=0,1,2,3$: $k=0$: 30°, $k=1$: 120°, $k=2$: 210°, $k=3$: 300° **The four 4th roots** are: $2\text{cis}(30^\circ)$, $2\text{cis}(120^\circ)$, $2\text{cis}(210^\circ)$, $2\text{cis}(300^\circ)$
7) The magnitude of each root is the cube root of 1, which is 1. The angles are $\frac{270^\circ+360^\circ k}{3}$ for $k=0,1,2$: $k=0$: 90°, $k=1$: 210°, $k=2$: 330° **The three cube roots** are: $\text{cis}(90^\circ)$, $\text{cis}(210^\circ)$, $\text{cis}(330^\circ)$	**8)** First, convert to polar coordinates: - Magnitude: $r=\sqrt{1^2+(-\sqrt{3})^2}=2$ - Angle: $\theta=\arctan\left(\frac{-\sqrt{3}}{1}\right)=-60^\circ$ or $-60^\circ+360^\circ=300^\circ$ The magnitude of each root is the 4th root of 2, which is $\sqrt[4]{2}$. The angles are $\frac{300^\circ+360^\circ k}{4}$ for $k=0,1,2,3$: $k=0$: 75°, $k=1$: 165°, $k=2$: 255°, $k=3$: 345° **The four 4th roots are**: $\sqrt[4]{2}\text{cis}(75^\circ)$, $\sqrt[4]{2}\text{cis}(165^\circ)$, $\sqrt[4]{2}\text{cis}(255^\circ)$, $\sqrt[4]{2}\text{cis}(345^\circ)$

#3-16. Trigonometric Equations and Identities Practice

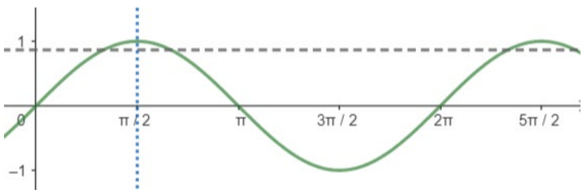
Solve for x, where x: $x \in [0^\circ, 360^\circ)$

1) $\sin x - \frac{\sqrt{3}}{2} = 0$	2) $\tan x = \sqrt{3}$
3) $\sqrt{4\cos^2 x - 1} - 1 = 0$	4) $\lvert 2\sin x - 1 \rvert = 1$
5) $\cos 2x = -3\sin x - 1$	6) $3\tan^2 x + \sqrt{3}\tan x = 0$

Solutions:

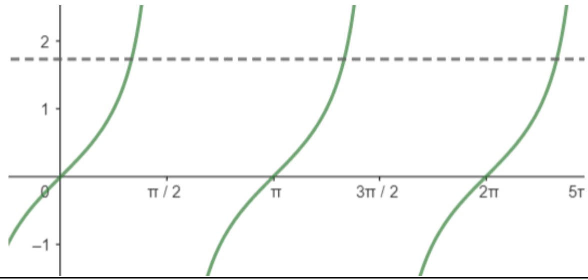
1) $\Rightarrow x = 60^\circ, 120^\circ$

- Solve: $\sin x = \dfrac{\sqrt{3}}{2}$
- $x = 60^\circ, 120^\circ$



2) $\Rightarrow x = 60^\circ, 240^\circ$

- Solve: $\tan x = \sqrt{3}$
- $x = 60^\circ, 240^\circ$



3) $\Rightarrow x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$

$\sqrt{4\cos^2 x - 1} - 1 = 0$

- Isolate the square root. $\sqrt{4\cos^2 x - 1} = 1$
- Square both sides to eliminate the square root. $4\cos^2 x - 1 = 1^2$
- Simplify the equation. $4\cos^2 x - 1 = 1$, $4\cos^2 x = 2$
- Divide by 4. $\cos^2 x = \dfrac{1}{2}$
- Take the square root of both sides. $\cos x = \pm\dfrac{\sqrt{2}}{2}$
- Find all x in the given interval. $x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$

4) $\Rightarrow x = 90^\circ,\ 0^\circ, 180^\circ$

$|2\sin x - 1| = 1$

- Solve the absolute value equation by considering both positive and negative values. $2\sin x - 1 = 1$ or $2\sin x - 1 = -1$
- Solve each equation.
  - For $2\sin x - 1 = 1$: $2\sin x = 2$, $\sin x = 1$, $x = 90^\circ$
  - For $2\sin x - 1 = -1$: $2\sin x = 0$, $\sin x = 0$, $x = 0^\circ, 180^\circ$

5) $\Rightarrow x = 210^\circ, -330^\circ$

$\cos 2x = -3\sin x - 1$

- Use the double-angle identity for cosine. $\Rightarrow \cos 2x = 2\cos^2 x - 1$
- Substitute the identity into the equation. $\Rightarrow 2\cos^2 x - 1 = -3\sin x - 1$
- Use the Pythagorean identity $\sin^2 x + \cos^2 x = 1$ to replace $\cos^2 x = 1 - \sin^2 x$.
- $2(1 - \sin^2 x) - 1 = -3\sin x - 1$
  $\Rightarrow 0 = 2\sin^2 x - 3\sin x - 2$
  $\Rightarrow (2\sin x + 1)(\sin x - 2) = 0$
- Solve: $2\sin x + 1 = 0 \Rightarrow \sin x = -\dfrac{1}{2}$
- $x = 210^\circ, -330^\circ$ (note: $\sin x - 2 = 0 \Rightarrow \sin x = 2$ is out of range)

6) $\Rightarrow x = 0^\circ, 180^\circ, 240^\circ, 300^\circ$

$3\tan^2 x + \sqrt{3}\tan x = 0$

- Factor: $\tan x(3\tan x + \sqrt{3}) = 0$
- $\tan x = 0$ or $3\tan x + \sqrt{3} = 0$
  - For $\tan x = 0$: $x = 0^\circ, 180^\circ$
  - For $3\tan x + \sqrt{3} = 0$: $\tan x = -\dfrac{\sqrt{3}}{3}$
    $x = 240^\circ, 300^\circ$

Chapter 4. Precalculus Concepts

#4-01. Exponential and Logarithmic Equations Practice

Find proper x value(s)

1) $16^{5-x} = 4^{x-2}$	2) $\sqrt{\frac{32^{x+2}}{8^{x}}} = 4^{x-1}$
3) $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 2 = 0$	4) $2^{x} = 16^{y-1}$ *and* $8^{y} = 2^{x-2}$
5) $\frac{1}{x} - \frac{1}{y} = 1$ *and* $\frac{2}{x} + \frac{3}{y} = 12$	6) $\frac{1}{x} - \frac{1}{\sqrt{x}} = 2$

Solutions:

1) $\Rightarrow x=4$ - Express both sides with a base of 2: $2^{4(5-x)}=2^{2(x-2)}$. - Equate the exponents: $4(5-x)=2(x-2)$. - Solve for x: . $20-4x=2x-4$ .. • $6x=24$. • $x=4$.	**2)** $\Rightarrow$ **No solution**: - Express all terms with a base of 2: $\sqrt{\frac{2^{5(x+2)}}{2^{3x}}}=2^{2(x-1)}$. - Simplify the square root and equate exponents: $2^{5(x+2)-3x}=2^{2(x-1)}$. • $2^{5x+10-3x}=2^{2x-2}$. • $2^{2x+10}=2^{2x-2}$. - Equate the exponents: $2x+10=2x-2$. - **No solution**, as the equation is not true for any x.
3) $\Rightarrow x=-1$ or $x=8$ - Let $u=x^{\frac{1}{3}}$. - The equation becomes $u^2-u-2=0$. - Factor: $(u-2)(u+1)=0$. - Solve for u: $u=2$ or $u=-1$. - Convert back to x: $x=2^3=8$ or $x=(-1)^3=-1$.	**4)** $\Rightarrow x=20,\ y=6$ - Express both equations with a base of 2: $2^x=2^{4(y-1)}$ and $2^{3y}=2^{x-2}$. - Equate the exponents: $x=4y-4$ and $3y=x-2$. - Solve the system of equations for x and y. - $x=20,\ y=6$
5) $\Rightarrow x=\frac{1}{3},\ y=\frac{1}{2}$ - Multiply the first equation by x and y: $y-x=xy$. - Multiply the second equation by x and y: $2y+3x=12xy$. - Solve the system of equations for x and y. - $x=\frac{1}{3},\ y=\frac{1}{2}$	**6)** $\Rightarrow x=\frac{1}{4}$ - Let $u=\sqrt{x}$. - The equation becomes $\frac{1}{u^2}-\frac{1}{u}=2$. - Multiply by u^2: $1-u=2u^2$. - Rearrange: $2u^2+u-1=0$. - Factor or use the quadratic formula to solve for u. $(2u-1)(u+1)=0\Rightarrow u=\frac{1}{2}$ *or* -1 - $u=\sqrt{x}=\frac{1}{2}$ *or* $-1\Rightarrow x=\frac{1}{4}$ (only solution, since $\sqrt{x}\neq-1$)

#4-02. Recursive Sequences and Inequality Solutions

Solve

1) If $x_1 = 2$ and $\frac{x_{n+1}}{x_n} = \sqrt{x_n - 1}$, then what is x_4?	2) If $2\sqrt{x+1} = \sqrt{8x}$, then what is x?
3) Find the set of values satisfying the inequality $\left\lvert \frac{4-2x}{3} \right\rvert \le 2$	4) Find the set of values satisfying the inequality $1 \le \lvert 2-x \rvert \le 2$

#4-20. Exponential Functions in Biological and Chemical Processes

Solve

1) A certain chemical decays at a rate such that its mass is reduced by one-third every 6 years. If the initial mass of the chemical is 150 grams, which expression shows the number of grams remaining after t years?	2) A population of algae doubles every 15 minutes. If $A(t)$ represents the number of algae after t minutes have elapsed, which equation can be used to model the exponential growth of the algae?
3) A sample of a radioactive isotope has an initial mass of 80 grams, and its mass decreases by 20% every 5 years. Which expression shows the number of grams remaining after t years?	4) A colony of fungi quadruples every 30 minutes. If $F(t)$ represents the number of fungi after t minutes have elapsed, which equation can be used to model the exponential growth of the fungi?
5) When exposed to sunlight, the number of bacteria in a culture decreases exponentially at the rate of 8% per hour. What is the best approximation for the number of hours required for the initial number of bacteria to decrease by 60%?	6) Given a starting population of 200 bacteria, the function $b(t)=200(3^t)$ can be used to find the number of bacteria, b, after t periods of time. If the length of each period is 10 minutes, how many minutes will it take for the bacteria population to reach 145,800?

Solutions:

1) $\Rightarrow M(t)=150\left(\frac{2}{3}\right)^{\frac{t}{6}}$ The mass of the chemical decays to two-thirds of its previous mass every 6 years. So, after t years, the mass remaining can be modeled by an exponential decay function: $M(t)=150\left(\frac{2}{3}\right)^{\frac{t}{6}}$	**2)** $\Rightarrow A(t)=2000(2)^{\frac{t}{15}}$ The population of algae doubles every 15 minutes, so the growth can be modeled by: $A(t)=2000(2)^{\frac{t}{15}}$
3) $\Rightarrow M(t)=80(0.8)^{\frac{t}{5}}$ The mass decreases by 20% every 5 years, which means it retains 80% of its mass every 5 years. The expression for the remaining mass after t years is: $M(t)=80(0.8)^{\frac{t}{5}}$	**4)** $\Rightarrow F(t)=1000(4)^{\frac{t}{30}}$ The colony of fungi quadruples every 30 minutes, so the growth can be modeled by: $F(t)=1000(4)^{\frac{t}{30}}$
5) $\Rightarrow t\approx 7.2$ To solve this, we use the exponential decay formula: $N(t)=N_0\cdot(1-r)^t$ where N_0 is the initial number, r is the decay rate per hour, and t is the time in hours. We want to find t when $N(t)=0.4N_0$ (since a 60% decrease means 40% remains). $0.4N_0=N_0\cdot(1-0.08)^t \Rightarrow 0.4=(0.92)^t$ Taking the natural logarithm of both sides: $\ln(0.4)=t\cdot\ln(0.92)\Rightarrow t=\frac{\ln(0.4)}{\ln(0.92)}$ Using a calculator: $t\approx 7.2$ So the best approximation is 7.2 hours.	**6)** $\Rightarrow 60$ minutes. We want to solve for t when $b(t)=145{,}800$. $\Rightarrow 145{,}800=200(3^t)\Rightarrow 729=3^t$ Since $729=3^6$, we have: $t=6$ Each period is 10 minutes, so the total time in minutes is $6\times 10=60$ minutes.

Chapter 5. Precalculus Combined Concepts

#5-01. Polynomial Division and Coefficient Determination

Solve.

1) If a polynomial $f(x) = x^2 + 3x + 4$ is divided by $(x-3)$, what is the remainder?	2) If a polynomial $f(x) = x^3 + 2x^2 + 3x + 4$ is divided by $(x+1)(x-2)$, then what is the remainder?
3) If a polynomial $f(x) = x^3 + ax^2 + bx + 1$ has a factor of $(x^2 - 1)$, what are the values of a and b?	4) $a(x+1) - b(x-4) = x + 6$ is true for all real values of x, what are the values of a and b?

Solutions:

1) $\Rightarrow$ 22 - Use the Remainder Theorem, which states that the remainder of a polynomial $f(x)$ divided by $(x-c)$ is $f(c)$. - Evaluate $f(3)$: $f(3)=3^2+3(3)+4=9+9+4=22$. - The remainder is 22.	**2)** $\Rightarrow 28x-50$ - Apply synthetic division or long division to divide $f(x)$ by $(x+1)(x-2)$. - First, divide by $(x+1)$ and then by $(x-2)$, or vice versa, and find the remainder for each. - Alternatively, find $f(-1)$ and $f(2)$ to use as coefficients in the linear remainder $Ax+B$. - Evaluate $f(-1)=(-1)^3+2(-1)^2+3(-1)+4$ $=-1+2-3+4=2$. - Evaluate $f(2)=(2)^3+2(2)^2+3(2)+4$ $=8+8+6+4=26$. - The remainder is of the form $A(x+1)+B(x-2)$, where $A=f(-1)$ and $B=f(2)$. - The remainder is $2(x+1)+26(x-2)$, which simplifies to $(2x+2)+(26x-52)=28x-50$.
3) $\Rightarrow a=-1$ and $b=-1$. - Since (x^2-1) is a factor, it means $f(1)=0$ and $f(-1)=0$. - Evaluate $f(1)=1+a+b+1=0 \Rightarrow a+b+2=0$. - Evaluate $f(-1)=-1+a-b+1=0 \Rightarrow a-b=0$. - Solve the system of equations: • $a+b=-2$ (from $f(1)=0$) • $a-b=0$ (from $f(-1)=0$) - Adding both equations gives $2a=-2$, so $a=-1$. - Substituting a into the second equation gives $-1-b=0$, so $b=-1$. - The values are $a=-1$ and $b=-1$.	**4)** $\Rightarrow a=2$ and $b=1$. - Expand both sides: $ax+a-bx+4b=x+6$. - Combine like terms: $(a-b)x+(a+4b)=x+6$. - For the equation to hold for all x, the coefficients of x must be equal, and the constants must be equal: • $a-b=1$ (coefficient of x) • $a+4b=6$ (constant term) - Solve the system of equations: • $a=b+1$ • $b+1+4b=6$ - Simplify the second equation: $5b+1=6$, so $b=1$. - Substitute b into the first equation: $a=1+1=2$. - The values are $a=2$ and $b=1$.

#5-20. Applications of Statistics in Real-Life Situations

- Use statistical methods to analyze and make decisions based on data.

Solve:

1) What is the mean, and how do you calculate it from a set of numbers?	2) What is the purpose of using statistics in weather forecasting?
3) How can statistics help in making decisions in business?	4) What is the median, and how is it different from the mean?
5) Explain how statistics can be used in healthcare to improve patient outcomes.	6) What is a statistical hypothesis, and how is it tested?
7) How do statistics support decision-making in environmental conservation efforts?	8) Discuss the role of statistical analysis in quality control and improvement in manufacturing.

Solutions:

1) What is the mean, and how do you calculate it from a set of numbers? - The mean is the average of a set of numbers, calculated by adding all the numbers together and then dividing by the count of the numbers. - For example, for the set 2, 4, 6, 8, the mean is $(2+4+6+8)/4=20/4=5$.	**2) What is the purpose of using statistics in weather forecasting?** - Statistics are used in weather forecasting to analyze historical weather data, identify patterns, and make predictions about future weather conditions based on those patterns.
3) How can statistics help in making decisions in business? - Statistics help in business decision-making by providing data analysis on market trends, customer behavior, and operational performance, enabling businesses to make informed decisions regarding marketing strategies, product development, and resource allocation.	**4) What is the median, and how is it different from the mean?** - The median is the middle value in a set of numbers ordered from smallest to largest. It is different from the mean as it is not influenced by extremely high or low values. - For example, in the set 1, 2, 3, 100, the median is 2.5 (the average of the two middle numbers, 2 and 3), while the mean is 26.5, significantly affected by the value 100.
5) Explain how statistics can be used in healthcare to improve patient outcomes. - Statistics in healthcare can be used to analyze patient data, identify trends in diseases, evaluate the effectiveness of treatments, and predict health outcomes. - This information helps healthcare providers make evidence-based decisions, leading to improved patient care and outcomes.	**6) What is a statistical hypothesis, and how is it tested?** - A statistical hypothesis is an assumption about a population parameter. It is tested using statistical methods to determine whether there is enough evidence to reject the hypothesis. - The process involves collecting data, calculating a test statistic, and comparing it to a critical value to decide if the observed data significantly deviates from what the hypothesis predicts.
7) How do statistics support decision-making in environmental conservation efforts? - Statistics support environmental conservation by analyzing data on pollution levels, species populations, and habitat conditions, among other factors. This analysis helps identify trends, assess the effectiveness of conservation measures, and make informed decisions on where to focus efforts and resources for maximum impact.	**8) Discuss the role of statistical analysis in quality control and improvement in manufacturing.** - Statistical analysis in manufacturing quality control involves collecting and analyzing data on production processes and product quality. - Techniques such as control charts and process capability analysis help identify variations in the process and deviations from quality standards. - This information is used to make adjustments to the process, reduce variability, and improve product quality, leading to higher efficiency, reduced waste, and increased customer satisfaction.